1. Products and Factors
2. Geometrical Optics - Grade 10
3. Force, Momentum and Impulse
4. Gravity and Mechanical Energy
5. 2D and 3D Wavefronts
6. Colour
7. Electromagnetic Radiation

Products and Factors

Introduction

In this chapter you will learn how to work with algebraic expressions. You will recap some of the work on factorisation and multiplying out expressions that you learnt in earlier grades. This work will then be extended upon for Grade 10.

Recap of Earlier Work

The following should be familiar. Examples are given as reminders.

Parts of an Expression

Mathematical expressions are just like sentences and their parts have special names. You should be familiar with the following names used to describe the parts of a mathematical expression.
Equation:

$$a \cdot x^k + b \cdot x + c^m = 0$$
$$d \cdot y^p + e \cdot y + f \leq 0$$

Name	**Examples (separated by commas)**
term	$a \cdot x^k$,$b \cdot x$, c^m, $d \cdot y^p$, $e \cdot y$, f
expression	$a \cdot x^k + b \cdot x + c^m$, $d \cdot y^p + e \cdot y + f$
coefficient	a, b, d, e
exponent (or index)	k, p
base	x, y, c
constant	a, b, c, d, e, f
variable	x, y
equation	$a \cdot x^k + b \cdot x + c^m = 0$
inequality	$d \cdot y^p + e \cdot y + f \leq 0$
binomial	expression with two terms

trinomial	expression with three terms

Product of Two Binomials

A *binomial* is a mathematical expression with two terms, e.g. $(ax+b)$ and $(cx+d)$. If these two binomials are multiplied, the following is the result:
Equation:

$$\begin{aligned}(a\cdot x+b)(c\cdot x+d) &= (ax)(c\cdot x+d)+b(c\cdot x+d)\\ &= (ax)(cx)+(ax)d+b(cx)+b\cdot d\\ &= ax^2+x\left(ad+bc\right)+bd\end{aligned}$$

Exercise:
Product of two binomials

Problem: Find the product of $(3x-2)(5x+8)$

Solution:

Multiply out and solve

Equation:

$$\begin{aligned}(3x-2)(5x+8) &= (3x)(5x)+(3x)(8)+(-2)(5x)+(-2)(8)\\ &= 15x^2+24x-10x-16\\ &= 15x^2+14x-16\end{aligned}$$

The product of two identical binomials is known as the *square of the binomial* and is written as:
Equation:

$$(ax+b)^2=a^2x^2+2abx+b^2$$

If the two terms are $ax+b$ and $ax-b$ then their product is:
Equation:

$$\left(ax+b\right)\left(ax-b\right)=a^2x^2-b^2$$

This is known as the *difference of two squares*.

Factorisation

Factorisation is the opposite of expanding brackets. For example expanding brackets would require $2(x+1)$ to be written as $2x+2$. Factorisation would be to start with $2x+2$ and to end up with

$2(x+1)$. In previous grades, you factorised based on common factors and on difference of squares.

Common Factors

Factorising based on common factors relies on there being common factors between your terms. For example, $2x-6x^2$ can be factorised as follows:
Equation:

$$2x-6x^2=2x\left(1-3x\right)$$

Investigation : Common Factors

Find the highest common factors of the following pairs of terms:

(a) $6y;18x$	(b) $12mn;8n$	(c) $3st;4su$	(d) $18kl;9kp$	(e) $abc;ac$
(f) $2xy;4xyz$	(g) $3uv;6u$	(h) $9xy;15xz$	(i) $24xyz;16yz$	(j) $3m;45n$

Difference of Two Squares

We have seen that:
Equation:

$$\left(ax+b\right)\left(ax-b\right)=a^2x^2-b^2$$

Since [link] is an equation, both sides are always equal. This means that an expression of the form:
Equation:

$$a^2x^2-b^2$$

can be factorised to
Equation:

$$(ax+b)(ax-b)$$

Therefore,
Equation:

$$a^2x^2-b^2=\left(ax+b\right)\left(ax-b\right)$$

For example, $x^2 - 16$ can be written as $\left(x^2 - 4^2\right)$ which is a difference of two squares. Therefore, the factors of $x^2 - 16$ are $(x - 4)$ and $(x + 4)$.

Exercise:
Factorisation

Problem: Factorise completely: $b^2y^5 - 3aby^3$

Solution:

Find the common factors:

Equation:

$$b^2y^5 - 3aby^3 = by^3\left(by^2 - 3a\right)$$

Exercise:
Factorising binomials with a common bracket

Problem: Factorise completely: $3a(a - 4) - 7(a - 4)$

Solution:

Find the common factors

$(a - 4)$is the common factor

Equation:

$$3a(a - 4) - 7(a - 4) = (a - 4)(3a - 7)$$

Exercise:
Factorising using a switch around in brackets

Problem: Factorise $5(a - 2) - b(2 - a)$

Solution:

Note that

Equation:

$$\begin{aligned} 5(a - 2) - b(2 - a) &= 5(a - 2) - [-b(a - 2)] \\ &= 5(a - 2) + b(a - 2) \\ &= (a - 2)(5 + b) \end{aligned}$$

Recap

1. Find the products of:

(a) $2y(y+4)$	(b) $(y+5)(y+2)$	(c) $(y+2)(2y+1)$
(d) $(y+8)(y+4)$	(e) $(2y+9)(3y+1)$	(f) $(3y-2)(y+6)$

[missing_resource: http://www.fhsst.org/lxl]

2. Factorise:

   1. $2l+2w$
   2. $12x+32y$
   3. $6x^2+2x+10x^3$
   4. $2xy^2+xy^2z+3xy$
   5. $-2ab^2-4a^2b$

   [missing_resource: http://www.fhsst.org/lqV]

3. Factorise completely:

(a) $7a+4$	(b) $20a-10$	(c) $18ab-3bc$
(d) $12kj+18kq$	(e) $16k^2-4k$	(f) $3a^2+6a-18$
(g) $-6a-24$	(h) $-2ab-8a$	(i) $24kj-16k^2j$
(j) $-a^2b-b^2a$	(k) $12k^2j+24k^2j^2$	(l) $72b^2q-18b^3q^2$
(m) $4(y-3)+k(3-y)$	(n) $a(a-1)-5(a-1)$	(o) $bm(b+4)-6m(b+4)$
(p) $a^2\left(a+7\right)+a\left(a+7\right)$	(q) $3b(b-4)-7(4-b)$	(r) $a^2b^2c^2-1$

[missing_resource: http://www.fhsst.org/lqE]

More Products

Khan Academy video on products of polynomials.
[missing_resource: http://www.youtube.com/v/fGThIRpWEE4&rel=0]

We have seen how to multiply two binomials in "Product of Two Binomials". In this section, we learn how to multiply a binomial (expression with two terms) by a trinomial (expression with three terms). Fortunately, we use the same methods we used to multiply two binomials to multiply a binomial and a trinomial.

For example, multiply $2x+1$ by x^2+2x+1.

Equation:

$$\begin{aligned} & (2x+1)\left(x^2+2x+1\right) \\ = \; & 2x\left(x^2+2x+1\right)+1\left(x^2+2x+1\right) \quad \text{(apply distributive law)} \\ = \; & \left[2x\left(x^2\right)+2x\left(2x\right)+2x\left(1\right)\right]+\left[1\left(x^2\right)+1\left(2x\right)+1\left(1\right)\right] \\ = \; & 2x^3+4x^2+2x+x^2+2x+1 \quad \text{(expand the brackets)} \\ = \; & 2x^3+\left(4x^2+x^2\right)+\left(2x+2x\right)+1 \quad \text{(group like terms to simplify)} \\ = \; & 2x^3+5x^2+4x+1 \quad \text{(simplify to get final answer)} \end{aligned}$$

Note: Multiplication of Binomial with Trinomial

If the binomial is $A+B$ and the trinomial is $C+D+E$, then the very first step is to apply the distributive law:

Equation:

$$(A+B)(C+D+E)=A(C+D+E)+B(C+D+E)$$

If you remember this, you will never go wrong!

Exercise:

Multiplication of Binomial with Trinomial

Problem: Multiply $x-1$ with x^2-2x+1.

Solution:

Determine what is given and what is required

We are given two expressions: a binomial, $x-1$, and a trinomial, x^2-2x+1. We need to multiply them together.

Determine how to approach the problem

Apply the distributive law and then simplify the resulting expression.

Solve the problem

Equation:

$$\begin{aligned}
& (x-1)\left(x^2-2x+1\right) \\
= \ & x\left(x^2-2x+1\right)-1\left(x^2-2x+1\right) \quad \text{(apply distributive law)} \\
= \ & \left[x\left(x^2\right)+x(-2x)+x(1)\right]+\left[-1\left(x^2\right)-1(-2x)-1(1)\right] \\
= \ & x^3-2x^2+x-x^2+2x-1 \quad \text{(expand the brackets)} \\
= \ & x^3+\left(-2x^2-x^2\right)+(x+2x)-1 \quad \text{(group like terms to simplify)} \\
= \ & x^3-3x^2+3x-1 \quad \text{(simplify to get final answer)}
\end{aligned}$$

Write the final answer

The product of $x-1$ and x^2-2x+1 is x^3-3x^2+3x-1.

Exercise:
Sum of Cubes

Problem: Find the product of $x+y$ and x^2-xy+y^2.

Solution:

Determine what is given and what is required

We are given two expressions: a binomial, $x+y$, and a trinomial, x^2-xy+y^2. We need to multiply them together.

Determine how to approach the problem

Apply the distributive law and then simplify the resulting expression.

Solve the problem

Equation:

$$\begin{aligned}
& (x+y)\left(x^2-xy+y^2\right) \\
= \ & x\left(x^2-xy+y^2\right)+y\left(x^2-xy+y^2\right) \quad \text{(apply distributive law)} \\
= \ & \left[x\left(x^2\right)+x(-xy)+x\left(y^2\right)\right]+\left[y\left(x^2\right)+y(-xy)+y\left(y^2\right)\right] \\
= \ & x^3-x^2y+xy^2+yx^2-xy^2+y^3 \quad \text{(expand the brackets)} \\
= \ & x^3+\left(-x^2y+yx^2\right)+\left(xy^2-xy^2\right)+y^3 \quad \text{(group like terms to simplify)} \\
= \ & x^3+y^3 \quad \text{(simplify to get final answer)}
\end{aligned}$$

Write the final answer

The product of $x+y$ and x^2-xy+y^2 is x^3+y^3.

Note: We have seen that:
Equation:

$$(x+y)\left(x^2-xy+y^2\right)=x^3+y^3$$

This is known as a *sum of cubes*.

Investigation : Difference of Cubes

Show that the difference of cubes ($x^3 - y^3$) is given by the product of $x - y$ and $x^2 + xy + y^2$.

Products

1. Find the products of:

(a) $\left(-2y^2 - 4y + 11\right)\left(5y - 12\right)$	(b) $\left(-11y + 3\right)\left(-10y^2 - 7y - 9\right)$
(c) $\left(4y^2 + 12y + 10\right)\left(-9y^2 + 8y + 2\right)$	(d) $\left(7y^2 - 6y - 8\right)\left(-2y + 2\right)$
(e) $\left(10y^5 + 3\right)\left(-2y^2 - 11y + 2\right)$	(f) $\left(-12y - 3\right)\left(12y^2 - 11y + 3\right)$
(g) $\left(-10\right)\left(2y^2 + 8y + 3\right)$	(h) $\left(2y^6 + 3y^5\right)\left(-5y - 12\right)$
(i) $\left(6y^7 - 8y^2 + 7\right)\left(-4y - 3\right)\left(-6y^2 - 7y - 11\right)$	(j) $\left(-9y^2 + 11y + 2\right)\left(8y^2 + 6y - 7\right)$
(k) $\left(8y^5 + 3y^4 + 2y^3\right)\left(5y + 10\right)\left(12y^2 + 6y + 6\right)$	(l) $(-7y + 11)(-12y + 3)$
(m) $\left(4y^3 + 5y^2 - 12y\right)\left(-12y - 2\right)\left(7y^2 - 9y + 12\right)$	(n) $\left(7y + 3\right)\left(7y^2 + 3y + 10\right)$
(o) $\left(9\right)\left(8y^2 - 2y + 3\right)$	(p) $\left(-12y + 12\right)\left(4y^2 - 11y + 11\right)$
(q) $\left(-6y^4 + 11y^2 + 3y\right)\left(10y + 4\right)\left(4y - 4\right)$	(r) $\left(-3y^6 - 6y^3\right)\left(11y - 6\right)\left(10y - 10\right)$
(s) $\left(-11y^5 + 11y^4 + 11\right)\left(9y^3 - 7y^2 - 4y + 6\right)$	(t) $\left(-3y + 8\right)\left(-4y^3 + 8y^2 - 2y + 12\right)$

[missing_resource: http://www.fhsst.org/llz]

Factorising a Quadratic

Khan Academy video on factorising a quadratic.
[missing_resource: http://www.youtube.com/v/eF6zYNzlZKQ&rel=0]

Factorisation can be seen as the reverse of calculating the product of factors. In order to factorise a quadratic, we need to find the factors which when multiplied together equal the original quadratic.

Let us consider a quadratic that is of the form $ax^2 + bx$. We can see here that x is a common factor of both terms. Therefore, $ax^2 + bx$ factorises to $x(ax + b)$. For example, $8y^2 + 4y$ factorises to $4y(2y + 1)$.

Another type of quadratic is made up of the difference of squares. We know that:
Equation:

$$(a + b)\,(a - b) = a^2 - b^2.$$

This is true for any values of a and b, and more importantly since it is an equality, we can also write:
Equation:

$$a^2 - b^2 = (a + b)\,(a - b).$$

This means that if we ever come across a quadratic that is made up of a difference of squares, we can immediately write down what the factors are.
Exercise:
Difference of Squares

Problem: Find the factors of $9x^2 - 25$.

Solution:

Examine the quadratic

We see that the quadratic is a difference of squares because:

Equation:

$$(3x)^2 = 9x^2$$

and

Equation:

$$5^2 = 25.$$

Write the quadratic as the difference of squares

Equation:

$$9x^2 - 25 = (3x)^2 - 5^2$$

Write the factors

Equation:

$$(3x)^2 - 5^2 = (3x - 5)\,(3x + 5)$$

Write the final answer

The factors of $9x^2 - 25$ are $(3x - 5)(3x + 5)$.

These types of quadratics are very simple to factorise. However, many quadratics do not fall into these categories and we need a more general method to factorise quadratics like $x^2 - x - 2$?

We can learn about how to factorise quadratics by looking at how two binomials are multiplied to get a quadratic. For example, $(x + 2)(x + 3)$ is multiplied out as:
Equation:

$$\begin{aligned}(x+2)(x+3) &= x(x+3)+2(x+3)\\ &= (x)(x)+3x+2x+(2)(3)\\ &= x^2+5x+6.\end{aligned}$$

We see that the x^2 term in the quadratic is the product of the x-terms in each bracket. Similarly, the 6 in the quadratic is the product of the 2 and 3 in the brackets. Finally, the middle term is the sum of two terms.

So, how do we use this information to factorise the quadratic?

Let us start with factorising x^2+5x+6 and see if we can decide upon some general rules. Firstly, write down two brackets with an x in each bracket and space for the remaining terms.
Equation:

$$(\quad x \qquad)(\quad x \qquad)$$

Next, decide upon the factors of 6. Since the 6 is positive, these are:

Factors of 6	
1	6
2	3
-1	-6
-2	-3

Therefore, we have four possibilities:

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$

Next, we expand each set of brackets to see which option gives us the correct middle term.

Option 1	Option 2	Option 3	Option 4
$(x+1)(x+6)$	$(x-1)(x-6)$	$(x+2)(x+3)$	$(x-2)(x-3)$
x^2+7x+6	x^2-7x+6	x^2+5x+6	x^2-5x+6

We see that Option 3 (x+2)(x+3) is the correct solution. As you have seen that the process of factorising a quadratic is mostly trial and error, there is some information that can be used to simplify the process.

Method: Factorising a Quadratic

1. First, divide the entire equation by any common factor of the coefficients so as to obtain an equation of the form $ax^2+bx+c=0$ where a, b and c have no common factors and a is positive.
2. Write down two brackets with an x in each bracket and space for the remaining terms.
   Equation:

$$(\quad x \qquad)(\quad x \qquad)$$

3. Write down a set of factors for a and c.
4. Write down a set of options for the possible factors for the quadratic using the factors of a and c.
5. Expand all options to see which one gives you the correct answer.

There are some tips that you can keep in mind:

- If c is positive, then the factors of c must be either both positive or both negative. The factors are both negative if b is negative, and are both positive if b is positive. If c is negative, it means only one of the factors of c is negative, the other one being positive.
- Once you get an answer, multiply out your brackets again just to make sure it really works.

Exercise:
Factorising a Quadratic

Problem: Find the factors of $3x^2+2x-1$.

Solution:

Check whether the quadratic is in the form with positive.

The quadratic is in the required form.

Write down two brackets with an in each bracket and space

Equation:

$$(\quad x \qquad)(\quad x \qquad)$$

Write down a set of factors for a and c.

Write down a set of options for the possible

Option 1	Option 2

for the remaining terms.

The possible factors for a are: (1,3). The possible factors for c are: (-1,1) or (1,-1).

The factors of the quadratic using the factors of a and c. Therefore, there are two possible options.

$(x-1)(3x+1)$	$(x+1)(3x-1)$
$3x^2-2x-1$	$3x^2+2x-1$

Check your answer

Equation:

$$\begin{array}{rcl}(x+1)(3x-1) & = & x(3x-1)+1(3x-1)\\ & = & (x)(3x)+(x)(-1)+(1)(3x)+(1)(-1)\\ & = & 3x^2-x+3x-1\\ & = & x^2+2x-1.\end{array}$$

Write the final answer

The factors of $3x^2+2x-1$ are $(x+1)$ and $(3x-1)$.

Factorising a Trinomial

1. Factorise the following:

(a) $x^2+8x+15$	(b) $x^2+10x+24$	(c) x^2+9x+8
(d) $x^2+9x+14$	(e) $x^2+15x+36$	(f) $x^2+12x+36$

[missing_resource: http://www.fhsst.org/liY]

2. Factorise the following:

   a. $x^2-2x-15$
   b. x^2+2x-3
   c. x^2+2x-8
   d. x^2+x-20
   e. x^2-x-20

   [missing_resource: http://www.fhsst.org/lir]

3. Find the factors for the following trinomial expressions:
    a. $2x^2 + 11x + 5$
    b. $3x^2 + 19x + 6$
    c. $6x^2 + 7x + 2$
    d. $12x^2 + 8x + 1$
    e. $8x^2 + 6x + 1$

    [missing_resource: http://www.fhsst.org/li1]

4. Find the factors for the following trinomials:
    a. $3x^2 + 17x - 6$
    b. $7x^2 - 6x - 1$
    c. $8x^2 - 6x + 1$
    d. $2x^2 - 5x - 3$

    [missing_resource: http://www.fhsst.org/liC]

Factorisation by Grouping

One other method of factorisation involves the use of common factors. We know that the factors of $3x + 3$ are 3 and $(x + 1)$. Similarly, the factors of $2x^2 + 2x$ are $2x$ and $(x + 1)$. Therefore, if we have an expression:

Equation:

$$2x^2 + 2x + 3x + 3$$

then we can factorise as:

Equation:

$$2x(x + 1) + 3(x + 1).$$

You can see that there is another common factor: $x + 1$. Therefore, we can now write:

Equation:

$$(x + 1)(2x + 3).$$

We get this by taking out the $x + 1$ and seeing what is left over. We have a $+2x$ from the first term and a $+3$ from the second term. This is called *factorisation by grouping*.

Exercise:

Factorisation by Grouping

Problem: Find the factors of $7x + 14y + bx + 2by$ by grouping

---

Solution:

Determine if there are common factors to all

terms	There are no factors that are common to all terms.
Determine if there are factors in common between some terms	7 is a common factor of the first two terms and b is a common factor of the second two terms.
Re-write expression taking the factors into account	**Equation:** $7x+14y+bx+2by=7(x+2y)+b(x+2y)$
Determine if there are more common factors	$x+2y$ is a common factor.
Re-write expression taking the factors into account	**Equation:** $7(x+2y)+b(x+2y)=(x+2y)(7+b)$
Write the final answer	The factors of $7x+14y+bx+2by$ are $(7+b)$ and $(x+2y)$.

Khan Academy video on factorising a trinomial by grouping.
[missing_resource: http://www.youtube.com/v/HXIj16mjfgk&rel=0]

Factorisation by Grouping

1. Factorise by grouping: $6x+\mathrm{a}+2ax+3$
   [missing_resource: http://www.fhsst.org/lih]
2. Factorise by grouping: $x^2-6x+5x-30$
   [missing_resource: http://www.fhsst.org/liS]
3. Factorise by grouping: $5x+10y-ax-2ay$
   [missing_resource: http://www.fhsst.org/liJ]
4. Factorise by grouping: $a^2-2a-ax+2x$
   [missing_resource: http://www.fhsst.org/liu]
5. Factorise by grouping: $5xy-3y+10x-6$
   [missing_resource: http://www.fhsst.org/liz]

Simplification of Fractions

In some cases of simplifying an algebraic expression, the expression will be a fraction. For example,
Equation:

$$\frac{x^2+3x}{x+3}$$

has a quadratic in the numerator and a binomial in the denominator. You can apply the different factorisation methods to simplify the expression.
Equation:

$$\begin{aligned} & \frac{x^2+3x}{x+3} \\ = & \ \frac{x(x+3)}{x+3} \\ = & \ x \quad \text{provided x} \neq -3 \end{aligned}$$

If x were 3 then the denominator, $x-3$, would be 0 and the fraction undefined.

Exercise:
Simplification of Fractions

Problem: Simplify: $\frac{2x-b+x-ab}{ax^2-abx}$

Solution:

Factorise numerator and denominator

Use *grouping* for numerator and *common factor* for denominator in this example.

Equation:

$$\begin{aligned} & = \frac{(ax-ab)+(x-b)}{ax^2-abx} \\ & = \frac{a(x-b)+(x-b)}{ax(x-b)} \\ & = \frac{(x-b)(a+1)}{ax(x-b)} \end{aligned}$$

Cancel out same factors

The simplified answer is:

Equation:

$$= \frac{a+1}{ax}$$

Exercise:
Simplification of Fractions

Problem: Simplify: $\frac{x^2-x-2}{x^2-4} \div \frac{x^2+x}{x^2+2x}$

Solution:

Factorise numerators and denominators

Equation:

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \div \frac{x(x+1)}{x(x+2)}$$

Multiply by factorised reciprocal

Equation:

$$= \frac{(x+1)(x-2)}{(x+2)(x-2)} \times \frac{x(x+2)}{x(x+1)}$$

Cancel out same factors	The simplified answer is	**Equation:**	$= 1$

Simplification of Fractions

1. Simplify:

(a) $\frac{3a}{15}$	(b) $\frac{2a+10}{4}$
(c) $\frac{5a+20}{a+4}$	(d) $\frac{a^2-4a}{a-4}$
(e) $\frac{3a^2-9a}{2a-6}$	(f) $\frac{9a+27}{9a+18}$
(g) $\frac{6ab+2a}{2b}$	(h) $\frac{16x^2y-8xy}{12x-6}$
(i) $\frac{4xyp-8xp}{12xy}$	(j) $\frac{3a+9}{14} \div \frac{7a+21}{a+3}$
(k) $\frac{a^2-5a}{2a+10} \div \frac{3a+15}{4a}$	(l) $\frac{3xp+4p}{8p} \div \frac{12p^2}{3x+4}$
(m) $\frac{16}{2xp+4x} \div \frac{6x^2+8x}{12}$	(n) $\frac{24a-8}{12} \div \frac{9a-3}{6}$
(o) $\frac{a^2+2a}{5} \div \frac{2a+4}{20}$	(p) $\frac{p^2+pq}{7p} \div \frac{8p+8q}{21q}$
(q) $\frac{5ab-15b}{4a-12} \div \frac{6b^2}{a+b}$	(r) $\frac{f^2a-fa^2}{f-a}$

   [missing_resource: http://www.fhsst.org/lit]

2. Simplify: $\frac{x^2-1}{3} \times \frac{1}{x-1} - \frac{1}{2}$

   [missing_resource: http://www.fhsst.org/lie]

Summary

- Product of two binomials
- Factorising
- Distributive law
- Sum and difference of cubes

- Factorising quadratics
- Factorising by grouping
- Simplifying fractions

End of Chapter Exercises

1. Factorise:

(a) $a^2 - 9$	(b) $m^2 - 36$	(c) $9b^2 - 81$
(d) $16b^6 - 25a^2$	(e) $m^2 - (1/9)$	(f) $5 - 5a^2b^6$
(g) $16ba^4 - 81b$	(h) $a^2 - 10a + 25$	(i) $16b^2 + 56b + 49$
(j) $2a^2 - 12ab + 18b^2$	(k) $-4b^2 - 144b^8 + 48b^5$	

   [missing_resource: http://www.fhsst.org/liM]

2. Show that $(2x-1)^2 - (x-3)^2$ can be simplified to $(x+2)(3x-4)$

   [missing_resource: http://www.fhsst.org/lib]

3. What must be added to $x^2 - x + 4$ to make it equal to $(x+2)^2$

   [missing_resource: http://www.fhsst.org/liT]

Geometrical Optics - Grade 10

Introduction

You are indoors on a sunny day. A beam of sunlight through a window lights up a section of the floor. How would you draw this sunbeam? You might draw a series of parallel lines showing the path of the sunlight from the window to the floor. This is not exactly accurate – no matter how hard you look, you will not find unique lines of light in the sunbeam! However, this is a good way to draw light. It is also a good way to model light geometrically, as we will see in this chapter.

These narrow, imaginary lines of light are called **light rays**. Since light is an electromagnetic wave, you could think of a light ray as the path of a point on the crest of a wave. Or, you could think of a light ray as the path taken by a miniscule particle that carries light. We will always draw them the same way: as straight lines between objects, images, and optical devices.

We can use light rays to model mirrors, lenses, telescopes, microscopes, and prisms. The study of how light interacts with materials is **optics**. When dealing with light rays, we are usually interested in the shape of a material and the angles at which light rays hit it. From these angles, we can work out, for example, the distance between an object and its reflection. We therefore refer to this kind of optics as **geometrical optics**.

Light Rays

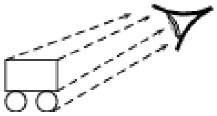
In physics we use the idea of a *light ray* to indicate the direction that light travels. Light rays are lines with arrows and are used to show the path that light travels. In [link], the light rays from the object enters the eye and the eye *sees* the object.

The most important thing to remember is that we can only see an object when light from the object enters our eyes. The object must be a source of light (for example a light bulb) or else it must reflect light from a source (for example the moon), and the reflected light enters our eyes.

Note: We cannot see an object unless light from that object enters our eyes.

Light ray

Light rays are straight lines with arrows to show the path of light.



We can only see an object when light from that object enters our eyes. We draw light as lines with arrows to show the direction the light travels. When the light travels from the object to the eye, the eye can see the object. Light rays entering the eye from the cart are shown as dashed lines. The second wheel of the cart will be invisible as no straight, unobstructed lines exist between it and the eye.

Note: Light rays are not an exact description of a general source of light. They are merely used to show the path that light travels.

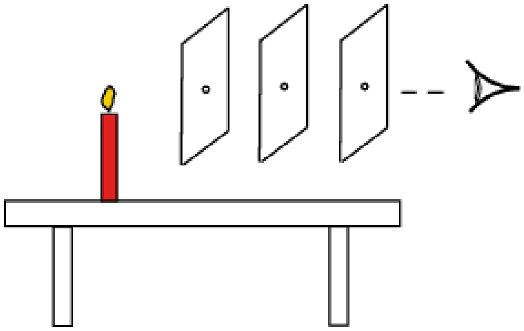
Investigation : Light travels in straight lines

Apparatus:

You will need a candle, matches and three sheets of paper.

Method:

1. Make a small hole in the middle of each of the three sheets of paper.
2. Light the candle.
3. Look at the burning candle through the hole in the first sheet of paper.
4. Place the second sheet of paper between you and the candle so that you can still see the candle through the holes.
5. Now do the same with the third sheet so that you can still see the candle. The sheets of paper must not touch each other.



Light travels in straight lines

6. What do you notice about the holes in the paper?

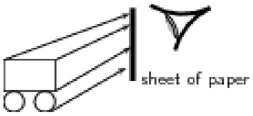
Conclusions:

In the investigation you will notice that the holes in the paper need to be in a straight line. This shows that light travels in a straight line. We cannot see around corners. This also proves that light does not bend around a corner, but travels straight.

Investigation : Light travels in straight lines

On a sunny day, stand outside and look at something in the distance, for example a tree, a flower or a car. From what we have learnt, we can see the tree, flower or car because light from the object is entering our eye. Now take a sheet of paper and hold it about 20 cm in front of your face. Can you still see the tree, flower or car? Why not?

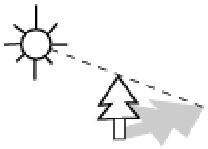
[link] shows that a sheet of paper in front of your eye prevents light rays from reaching your eye.



The sheet of paper prevents the light rays from reaching the eye, and the eye cannot see the object.

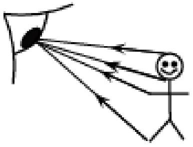
Shadows

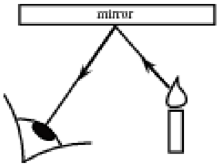
Objects cast shadows when light shines on them. This is more evidence that light travels in straight lines. The picture below shows how shadows are formed.



Ray Diagrams

A ray diagram is a drawing that shows the path of light rays. Light rays are drawn using straight lines and arrow heads. The figure below shows some examples of ray diagrams.





Light Rays

1. Give evidence to support the statement: “Light travels in straight lines”. Draw a ray diagram to prove this.
   Click here for the solution.

Reflection

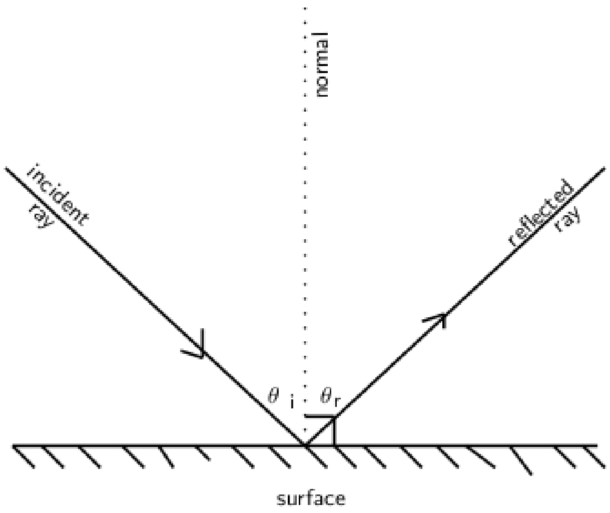
When you smile into a mirror, you see your own face smiling back at you. This is caused by the reflection of light rays on the mirror. Reflection occurs when a light ray bounces off a surface.

Terminology

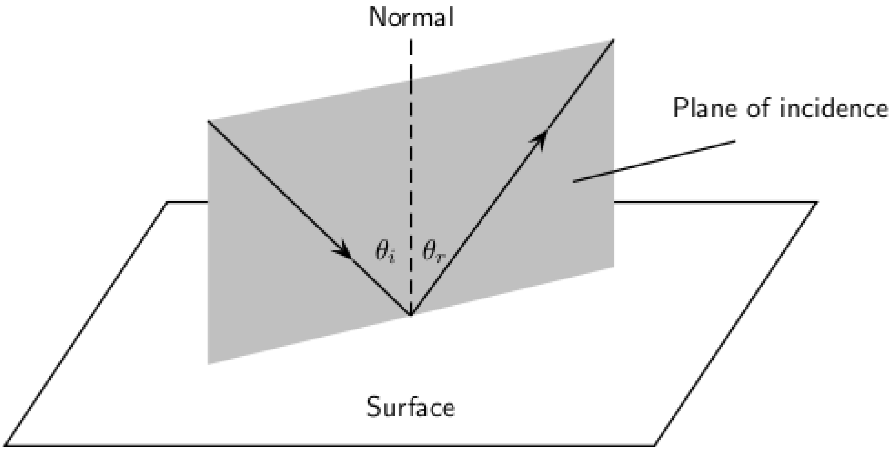
In Transverse Pulses and Transverse Waves we saw that when a pulse or wave strikes a surface it is *reflected*. This means that waves bounce off things. Sound waves bounce off walls, light waves bounce off mirrors, radar waves bounce off aeroplanes and it can explain how bats can fly at night and avoid things as thin as telephone wires. The phenomenon of reflection is a very important and useful one.

We will use the following terminology. The incoming light ray is called the **incident ray**. The light ray moving away from the surface is the **reflected ray**. The most important characteristic of these rays is their angles in relation to the reflecting surface. These angles are measured with respect to the normal of the surface. The **normal** is an imaginary line perpendicular to the surface. The **angle of incidence,** θ_i is measured between the incident ray and the surface normal. The **angle of reflection,** θ_r is measured between the reflected ray and the surface normal. This is shown in [link].

When a ray of light is reflected, the reflected ray lies in the same plane as the incident ray and the normal. This plane is called the **plane of incidence** and is shown in [link].



The angles of incidence and reflection are measured with respect to the surface normal.



The plane of incidence is the plane including the incident ray, reflected ray, and the surface normal.

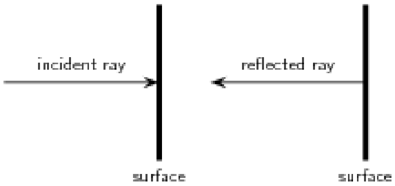
Law of Reflection

The **Law of Reflection** states that the angles of incidence and reflection are always equal and that the reflected ray always lies in the plane of incidence.

Law of Reflection

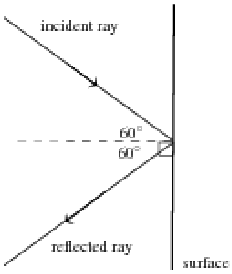
> The Law of Reflection states that the angle of incidence is equal to the angle of reflection.
> **Equation:**
>
> $$\theta_i = \theta_r$$

The simplest example of the law of incidence is if the angle of incidence is 0°. In this case, the angle of reflection is also 0°. You see this when you look straight into a mirror.



When a wave strikes a surface at right angles to the surface, then the wave is reflected directly back.

If the angle of incidence is not 0°, then the angle of reflection is also not 0°. For example, if a light strikes a surface at 60° to the surface normal, then the angle that the reflected ray makes with the surface normal is also 60° as shown in [link].



Ray diagram showing angle of incidence and angle of reflection. The Law of Reflection states that when a light ray reflects off a surface, the angle of reflection θ_r is the same as the angle of incidence θ_i.

Exercise: Law of Reflection

Problem:

An incident ray strikes a smooth reflective surface at an angle of 33° to the surface normal. Calculate the angle of reflection.

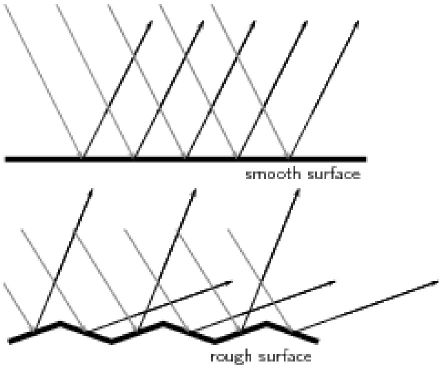
Solution:

Determine what is given and what is required	We are given the angle between the incident ray and the surface normal. This is the angle of incidence.	We are required to calculate the angle of reflection.
Determine how to approach the problem	We can use the Law of Reflection, which states that the angle of incidence is equal to the angle of reflection.	
Calculate the angle of reflection	We are given the angle of incidence to be 33°. Therefore, the angle of reflection is also 33°.	

Types of Reflection

The Law of Reflection holds for every light ray. Does this mean that when parallel rays approach a surface, the reflected rays will also be parallel? This depends on the texture of the reflecting surface.

Specular and diffuse reflection.



Specular Reflection

[link](a) shows a surface that is flat and even. Parallel incident light rays hit the smooth surface and parallel reflected light rays leave the surface. This type of reflection is called **specular reflection**. Specular reflection occurs when rays are reflected from a smooth, shiny surface. The normal to the surface is the same at every point on the surface. Parallel incident rays become parallel reflected rays. When you look in a mirror, the image you see is formed by specular reflection.

Diffuse Reflection

[link](b) shows a surface with bumps and curves. When multiple rays hit this uneven surface, **diffuse reflection** occurs. The incident rays are parallel but the reflected rays are not. Each point on the surface has a different normal. This means the angle of incidence is different at each point. Then according to the Law of Reflection, each angle of reflection is different. Diffuse reflection occurs when light rays are reflected from bumpy surfaces. You can still see a reflection as long as the surface is not too bumpy. Diffuse reflection enables us to see all objects that are not sources of light.

Experiment : Specular and Diffuse Reflection

A bouncing ball can be used to demonstrate the basic difference between specular and diffuse reflection.

Aim:

To demonstrate and compare specular and diffuse reflection.

Apparatus:

You will need:

1. a small ball (a tennis ball or a table tennis ball is perfect)
2. a smooth surface, like the floor inside the classroom
3. a very rough surface, like a rocky piece of ground

Method:

1. Bounce the ball on the smooth floor and observe what happens.
2. Bounce the ball on the rough ground floor and observe what happens.
3. What do you observe?
4. What is the difference between the two surfaces?

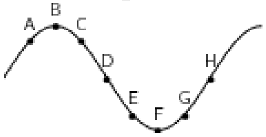
Conclusions:

You should have seen that the ball bounces (is reflected off the floor) in a predictable manner off the smooth floor, but bounces unpredictably on the rough ground.

The ball can be seen to be a ray of light and the floor or ground is the reflecting surface. For specular reflection (smooth surface), the ball bounces predictably. For diffuse reflection (rough surface), the ball bounces unpredictably.

Reflection

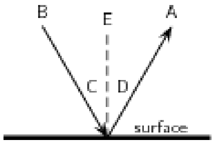
1. The diagram shows a curved surface. Draw normals to the surface at the marked points.



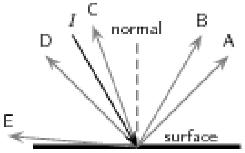
Click here for the solution.

2. Which of the points, A–H, in the diagram, correspond to the following:

    1. normal
    2. angle of incidence
    3. angle of reflection
    4. incident ray
    5. reflected ray

   Click here for the solution.
3. State the Law of Reflection. Draw a diagram, label the appropriate angles and write a mathematical expression for the Law of Reflection.
   Click here for the solution.
4. The diagram shows an incident ray I. Which of the other 5 rays (A, B, C, D, E) best represents the reflected ray of I?

   Click here for the solution.
5. A ray of light strikes a surface at 15° to the surface normal. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
   Click here for the solution.
6. A ray of light leaves a surface at 65° to the surface. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
   Click here for the solution.
7. Explain the difference between specular and diffuse reflection.
   Click here for the solution.
8. We see an object when the light that is reflected by the object enters our eyes. Do you think the reflection by most objects is specular reflection or diffuse reflection? Explain.
   Click here for the solution.
9. A beam of light (for example from a torch) is generally not visible at night, as it travels through air. Try this for yourself. However, if you shine the torch through dust, the beam is visible. Explain why this happens.
   Click here for the solution.

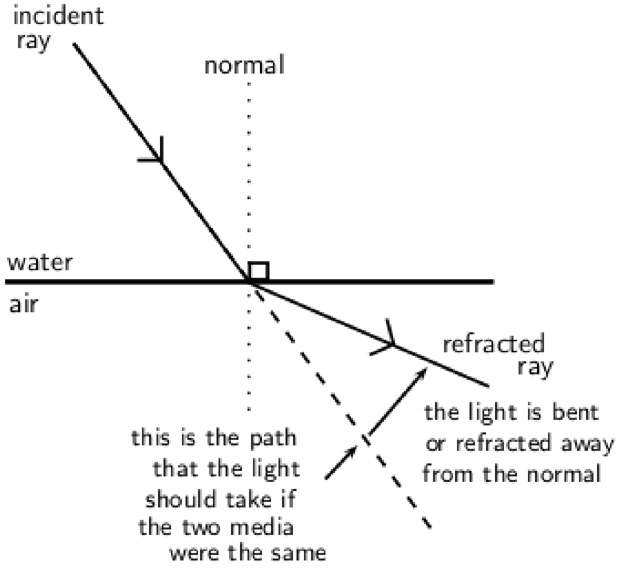
Refraction

In the previous sections we studied light reflecting off various surfaces. What happens when light passes *through* a medium? The speed of light, like that of all waves, is dependent on the medium in which it is travelling. When light moves from one medium into another (for example, from air to glass), the speed of light changes. The effect is that the light ray passing into a new medium is **refracted**, or bent. Refraction is therefore the bending of light as it moves from one optical medium to another.

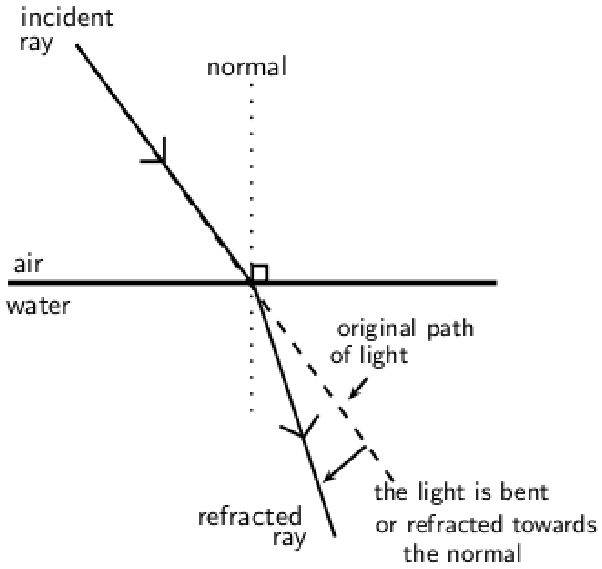
Refraction

> Refraction is the bending of light that occurs because light travels at different speeds in different materials.

When light travels from one medium to another, it will be bent away from its original path. When it travels from an optically dense medium like water or glass to a less dense medium like air, it will be refracted away from the normal ([link]). Whereas, if it travels from a less dense medium to a denser one, it will be refracted towards the normal ([link]).


incident ray
normal
water
air
refracted ray
the light is bent or refracted away from the normal
this is the path that the light should take if the two media were the same

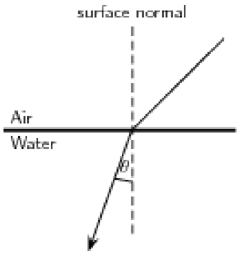
Light is moving from an optically dense medium to an optically less dense medium. Light is refracted away from the normal.

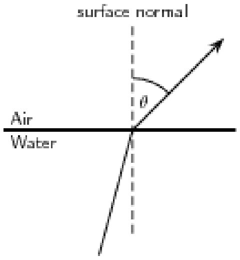


Light is moving from an optically less dense medium to an optically denser medium. Light is refracted towards the normal.

Just as we defined an angle of reflection in the previous section, we can similarly define an angle of refraction as the angle between the surface normal and the refracted ray. This is shown in [link].

Light moving from one medium to another bends towards or away from the surface normal. The angle of refraction θ is shown.





Refractive Index

Which is easier to travel through, air or water? People usually travel faster through air. So does light! The speed of light and therefore the degree of bending of the light depends on the *refractive index* of material through which the light passes. The refractive index (symbol n) is the ratio of the speed of light in a vacuum to its speed in the material.

Refractive Index

> The refractive index of a material is the ratio of the speed of light in a vacuum to its speed in the medium.

Note:
Interesting Fact
The symbol c is used to represent the speed of light in a vacuum.
Equation:

$$c = 299\ 792\ 485\,\mathrm{m \cdot s^{-1}}$$

For purposes of calculation, we use $3 \times 10^8\,\mathrm{m \cdot s^{-1}}$. A vacuum is a region with no matter in it, not even air. However, the speed of light in air is very close to that in a vacuum.

Refractive Index

The refractive index (symbol n) of a material is the ratio of the speed of light in a vacuum to its speed in the material and gives an indication of how difficult it is for light to get through the material.
Equation:

$$n = \frac{c}{v}$$

where

n =	refractive index (no unit)
c =	speed of light in a vacuum ($3,00 \times 10^{8}\,\mathrm{m \cdot s^{-1}}$)
v =	speed of light in a given medium ($\mathrm{m \cdot s^{-1}}$)

Refractive Index and Speed of Light

Using
Equation:

$$n = \frac{c}{v}$$

we can also examine how the speed of light changes in different media, because the speed of light in a vacuum (c) is constant.

If the refractive index n increases, the speed of light in the material v must decrease. Light therefore travels slowly through materials of high n.

[link] shows refractive indices for various materials. Light travels slower in any material than it does in a vacuum, so all values for n are greater than 1.

Medium	**Refractive Index**
Vacuum	1
Helium	1,000036
Air*	1,0002926
Carbon dioxide	1,00045
Water: Ice	1,31
Water: Liquid (20°C)	1,333
Acetone	1,36
Ethyl Alcohol (Ethanol)	1,36
Sugar solution (30%)	1,38
Fused quartz	1,46
Glycerine	1,4729
Sugar solution (80%)	1,49
Rock salt	1,516
Crown Glass	1,52
Sodium chloride	1,54
Polystyrene	1,55 to 1,59
Bromine	1,661
Sapphire	1,77
Glass (typical)	1,5 to 1,9

Cubic zirconia	2,15 to 2,18
Diamond	2,419
Silicon	4,01

Refractive indices of some materials. n_{air} is calculated at STP.

Snell's Law

Now that we know that the degree of bending, or the angle of refraction, is dependent on the refractive index of a medium, how do we calculate the angle of refraction?

The angles of incidence and refraction when light travels from one medium to another can be calculated using Snell's Law.

Snell's Law

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where

n_1 =	Refractive index of material 1
n_2 =	Refractive index of material 2
θ_1 =	Angle of incidence
θ_2 =	Angle of refraction

Remember that angles of incidence and refraction are measured from the normal, which is an imaginary line perpendicular to the surface.

Suppose we have two media with refractive indices n_1 and n_2. A light ray is incident on the surface between these materials with an **angle of incidence** θ_1. The refracted ray that passes through the second medium will have an **angle of refraction** θ_2.

Exercise:

Using Snell's Law

Problem:

A light ray with an angle of incidence of 35° passes from water to air. Find the angle of refraction using Snell's Law and [link]. Discuss the meaning of your answer.

Solution:

Determine the refractive indices of water and air

From [link], the refractive index is 1,333 for water and about 1 for air. We know the angle of incidence, so we are ready to use Snell's Law.

Substitute values

Equation:

According to Snell's Law:

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ 1,33 \sin 35^\circ &= 1 \sin\theta_2 \\ \sin\theta_2 &= 0,763 \\ \theta_2 &= 49,7^\circ \text{ or } 130,3^\circ \end{aligned}$$

Equation:

Since $130,3^\circ$ is larger than 90°, the solution is:

$$\theta_2 = 49,7^\circ$$

Discuss the answer

The light ray passes from a medium of high refractive index to one of low refractive index. Therefore, the light ray is bent away from the normal.

Exercise:

Using Snell's Law

Problem:

A light ray passes from water to diamond with an angle of incidence of 75°. Calculate the angle of refraction. Discuss the meaning of your answer.

Solution:

Determine the refractive indices of water and air

From [link], the refractive index is 1,333 for water and 2,42 for diamond. We know the angle of incidence, so we are ready to use Snell's Law.

Substitute values and solve

According to Snell's Law:

Equation:

$$\begin{aligned} n_1 \sin \theta_1 &= n_2 \sin \theta_2 \\ 1,33 \sin 75^\circ &= 2,42 \sin \theta_2 \\ \sin \theta_2 &= 0,531 \\ \theta_2 &= 32,1^\circ. \end{aligned}$$

Discuss the answer

The light ray passes from a medium of low refractive index to one of high refractive index. Therefore, the light ray is bent towards the normal.

If

Equation:

$$n_2 > n_1$$

then from Snell's Law,

Equation:

$$\sin \theta_1 > \sin \theta_2.$$

For angles smaller than 90°, $\sin \theta$ increases as θ increases. Therefore,

Equation:

$$\theta_1 > \theta_2.$$

This means that the angle of incidence is greater than the angle of refraction and the light ray is bent toward the normal.

Similarly, if

Equation:

$$n_2 < n_1$$

then from Snell's Law,
Equation:

$$\sin \theta_1 < \sin \theta_2.$$

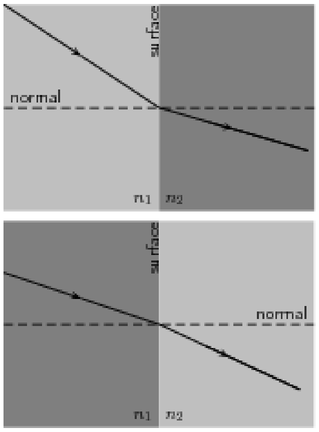
For angles smaller than 90°, $\sin \theta$ increases as θ increases. Therefore,
Equation:

$$\theta_1 < \theta_2.$$

This means that the angle of incidence is less than the angle of refraction and the light ray is away toward the normal.

Both these situations can be seen in [link].

Refraction of two light rays. (a) A ray travels from a medium of low refractive index to one of high refractive index. The ray is bent towards the normal. (b) A ray travels from a medium with a high refractive index to one with a low refractive index. The ray is bent away from the normal.



What happens to a ray that lies along the normal line? In this case, the angle of incidence is 0° and
Equation:

$$\begin{aligned} \sin\,\theta_2 &= \frac{n_1}{n_2}\sin\,\theta_1 \\ &= 0 \\ \therefore \theta_2 &= 0. \end{aligned}$$

This shows that if the light ray is incident at $0°$, then the angle of refraction is also $0°$. The ray passes through the surface unchanged, i.e. no refraction occurs.

Investigation : Snell's Law 1

The angles of incidence and refraction were measured in five unknown media and recorded in the table below. Use your knowledge about Snell's Law to identify each of the unknown media A - E. Use [link] to help you.

Medium 1	n_1	θ_1	θ_2	n_2	**Unknown Medium**
Air	1,000036	38	11,6	?	A
Air	1,000036	65	38,4	?	B
Vacuum	1	44	0,419	?	C
Air	1,000036	15	29,3	?	D
Vacuum	1	20	36,9	?	E

Investigation : Snell's Law 2

Zingi and Tumi performed an investigation to identify an unknown liquid. They shone a beam of light into the unknown liquid, varying the angle of incidence and recording the angle of refraction. Their results are recorded in the following table:

Angle of Incidence	Angle of Refraction
0,0°	0,00°
5,0°	3,76°
10,0°	7,50°
15,0°	11,2°
20,0°	14,9°
25,0°	18,5°
30,0°	22,1°
35,0°	25,5°
40,0°	28,9°
45,0°	32,1°
50,0°	35,2°
55,0°	38,0°
60,0°	40,6°
65,0°	43,0°
70,0°	?
75,0°	?
80,0°	?
85,0°	?

1. Write down an aim for the investigation.
2. Make a list of all the apparatus they used.

3. Identify the unknown liquid.
4. Predict what the angle of refraction will be for 70°, 75°, 80° and 85°.

Khan academy video on Snell's Law - 1
[missing_resource: http://www.youtube.com/v/HahjsBApxLE&rel=0]

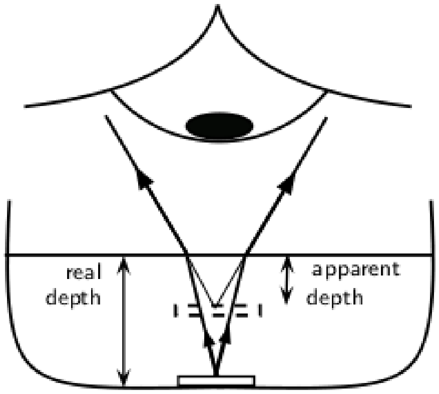
Apparent Depth

Imagine a coin on the bottom of a shallow pool of water. If you reach for the coin, you will miss it because the light rays from the coin are refracted at the water's surface.

Consider a light ray that travels from an underwater object to your eye. The ray is refracted at the water surface and then reaches your eye. Your eye does not know Snell's Law; it assumes light rays travel in straight lines. Your eye therefore sees the image of the at coin shallower location. This shallower location is known as the *apparent depth*.

The refractive index of a medium can also be expressed as
Equation:

$$n = \frac{\text{real depth}}{\text{apparent depth}}.$$



Exercise:
Apparent Depth 1

Problem:

A coin is placed at the bottom of a 40 cm deep pond. The refractive index for water is 1,33. How deep does the coin appear to be?

Solution:

Identify what is given and what is asked

n = 1,33

real depth = 40 cm

apparent depth = ?

Substitute values and find answer

Equation:

$$
\begin{aligned}
n &= \frac{\text{real depth}}{\text{apparent depth}} \\
1,33 &= \frac{40}{x} \\
x &= \frac{40}{1,33} = 30,08cm
\end{aligned}
$$

The coin appears to be 30,08 cm deep.

Exercise:
Apparent Depth 2

Problem:

A R1 coin appears to be 7 cm deep in a colourless liquid known to be listed in [link]. The depth of the liquid is 10,43 cm.

1. Determine the refractive index of the liquid.
2. Identify the liquid.

Solution:

Identify what is given and what is asked

real depth = 7 cm

apparent depth = 10,43 cm

n = ?

Identify the liquid.

Calculate refractive index

Equation:

$$n = \frac{\text{real depth}}{\text{apparent depth}} = \frac{10,43}{7} = 1,49$$

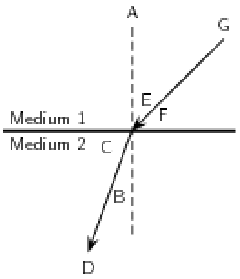
Identify the liquid

Use [link]. The liquid is an 80% sugar solution.

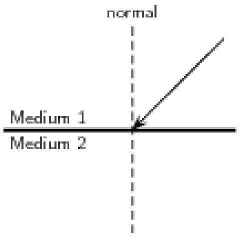
Refraction

1. Explain refraction in terms of a change of wave speed in different media.
   Click here for the solution.
2. In the diagram, label the following:

   a. angle of incidence
   b. angle of refraction
   c. incident ray
   d. refracted ray
   e. normal

   Click here for the solution.
3. What is refraction?
   Click here for the solution.
4. Describe what is meant by *the refractive index of a medium*.
   Click here for the solution.
5. State Snell's Law.
   Click here for the solution.
6. In the diagram, a ray of light strikes the interface between two media.



Draw what the refracted ray would look like if:

   a. medium 1 had a higher refractive index than medium 2.
   b. medium 1 had a lower refractive index than medium 2.

Click here for the solution.

7. Light travels from a region of glass into a region of glycerine, making an angle of incidence of 40°.

   a. Describe the path of the light as it moves into the glycerine.
   b. Calculate the angle of refraction.

   Click here for the solution.

8. A ray of light travels from silicon to water. If the ray of light in the water makes an angle of 69° to the surface normal, what is the angle of incidence in the silicon?

   Click here for the solution.

9. Light travels from a medium with $n = 1,25$ into a medium of $n = 1,34$, at an angle of 27° from the interface normal.

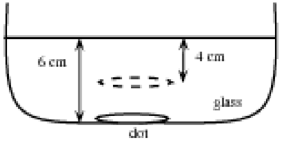
   a. What happens to the speed of the light? Does it increase, decrease, or remain the same?
   b. What happens to the wavelength of the light? Does it increase, decrease, or remain the same?
   c. Does the light bend towards the normal, away from the normal, or not at all?

   Click here for the solution.

10. Light travels from a medium with $n = 1,63$ into a medium of $n = 1,42$.

   a. What happens to the speed of the light? Does it increase, decrease, or remain the same?
   b. What happens to the wavelength of the light? Does it increase, decrease, or remain the same?
   c. Does the light bend towards the normal, away from the normal, or not at all?

Click here for the solution.

11. Light is incident on a glass prism. The prism is surrounded by air. The angle of incidence is 23°. Calculate the angle of reflection and the angle of refraction.
    Click here for the solution.
12. Light is refracted at the interface between air and an unknown medium. If the angle of incidence is 53° and the angle of refraction is 37°, calculate the refractive index of the unknown, second medium.
    Click here for the solution.
13. A coin is placed in a bowl of acetone (n = 1,36). The coin appears to be 10 cm deep. What is the depth of the acetone?
    Click here for the solution.
14. A dot is drawn on a piece of paper and a glass prism placed on the dot according to the diagram.
    

    Use the information supplied to determine the refractive index of glass.
    Click here for the solution.
15. Light is refracted at the interface between a medium of refractive index 1,5 and a second medium of refractive index 2,1. If the angle of incidence is 45°, calculate the angle of refraction.
    Click here for the solution.
16. A ray of light strikes the interface between air and diamond. If the incident ray makes an angle of 30° with the interface, calculate the angle made by the refracted ray with the interface.
    Click here for the solution.
17. **Challenge Question**: What values of n are physically impossible to achieve? Explain your answer. The values provide the limits of possible refractive indices.
    Click here for the solution.
18. **Challenge Question**: You have been given a glass beaker full of an unknown liquid. How would you identify what the liquid is? You have the following pieces of equipment available for the experiment: a laser, a protractor, a ruler, a pencil, and a reference guide containing optical properties of various liquids.
    Click here for the solution.

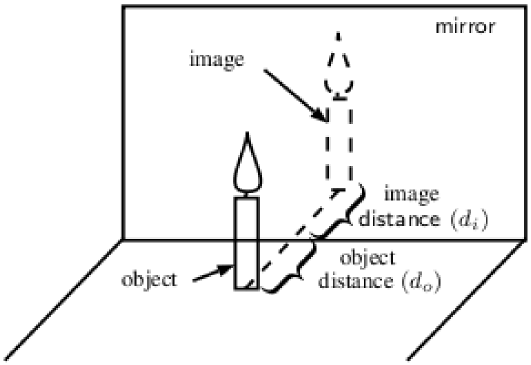
Mirrors

A mirror is a highly reflective surface. The most common mirrors are flat and are known as **plane mirrors**. Household mirrors are plane mirrors. They are made of a flat piece of glass with a thin layer of silver nitrate or aluminium on the back. However, other mirrors are curved and are either **convex mirrors** or are **concave mirrors**. The reflecting properties of all three types of mirrors will be discussed in this section.

Image Formation

Image

> An image is a representation of an object formed by a mirror or lens. Light from the image is seen.



An object formed in a mirror is real and upright.

If you place a candle in front of a mirror, you now see two candles. The actual, physical candle is called the object and the picture you see in the mirror is called the image. The **object** is the source of the incident rays. The **image** is the picture that is formed by the reflected rays.

The object could be an actual source that emits light, such as a light bulb or a candle. More commonly, the object reflects light from another source. When you look at your face in the mirror, your face does not emit light. Instead, light from a light bulb or from the sun reflects off your face and then hits the mirror. However, in working with light rays, it is easiest to pretend the light is coming from the object.

An image formed by reflection may be real or virtual. A **real** image occurs when light rays actually intersect at the image. A real image is inverted, or upside down. A **virtual** image occurs when light rays do not actually meet at the image. Instead, you "see" the image because your eye projects light rays backward. You are fooled into seeing an image! A virtual image is erect, or right side up (upright).

You can tell the two types apart by putting a screen at the location of the image. A real image can be formed on the screen because the light rays actually meet there. A virtual image cannot be seen on a screen, since it is not really there.

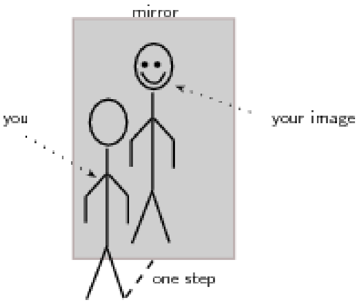
To describe objects and images, we need to know their locations and their sizes. The distance from the mirror to the object is the **object distance**, d_o.

The distance from the mirror to the image is the **image distance**, d_i.

Plane Mirrors

Investigation : Image formed by a mirror

1. Stand one step away from a large mirror
2. What do you observe in the mirror? This is called your image.
3. What size is your image? Bigger, smaller or the same size as you?
4. How far is your image from you? How far is your image from the mirror?
5. Is your image upright or upside down?
6. Take one step backwards. What does your image do? How far are you away from your image?
7. If it were a real object, which foot would the image of you right show fit?



An image in a

mirror is virtual,
upright, the same
size and inverted
front to back.

When you look into a mirror, you see an *image* of yourself.

The image created in the mirror has the following properties:

1. The image is *virtual*.
2. The image is the same distance behind the mirror as the object is in front of the mirror.
3. The image is inverted front to back.
4. The image is the same size as the object.
5. The image is upright.

Virtual images are images formed in places where light does not really reach. Light does not really pass through the mirror to create the image; it only appears to an observer as though the light were coming from behind the mirror. Whenever a mirror creates an image which is virtual, the image will always be located behind the mirror where light does not really pass.

Virtual Image

> A virtual image is upright, on the opposite side of the mirror as the object, and light does not actually reach it.

Ray Diagrams

We draw *ray diagrams* to predict the image that is formed by a plane mirror. A ray diagram is a geometrical picture that is used for analyzing the images formed by mirrors and lenses. We draw a few characteristic rays from the object to the mirror. We then follow ray-tracing rules to find the path of the rays and locate the image.

Note:A mirror obeys the Law of Reflection.

The ray diagram for the image formed by a plane mirror is the simplest possible ray diagram. [link] shows an object placed in front of a plane mirror. It is convenient to have a central line that runs perpendicular to the mirror. This imaginary line is called the **principal axis**.

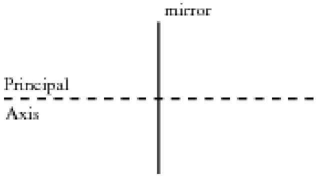
Note:
Ray diagrams
The following should be remembered when drawing ray diagrams:

1. Objects are represented by arrows. The length of the arrow represents the height of the object.
2. If the arrow points upwards, then the object is described as upright or erect. If the arrow points downwards then the object is described as inverted.
3. If the object is real, then the arrow is drawn with a solid line. If the object is virtual, then the arrow is drawn with a dashed line.

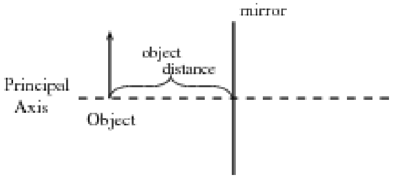
Method: Ray Diagrams for Plane Mirrors

Ray diagrams are used to find the position and size and whether the image is real or virtual.

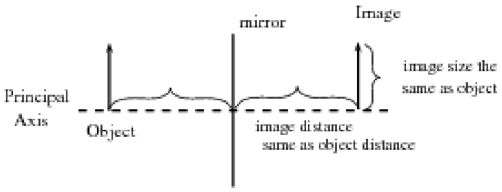
1. Draw the plane mirror as a straight line on a principal axis.



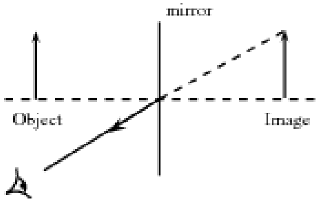
2. Draw the object as an arrow in front of the mirror.



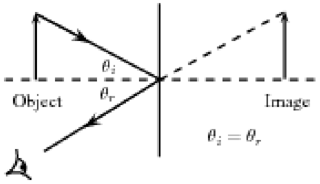
3. Draw the image of the object, by using the principle that the image is placed at the same distance behind the mirror that the object is in front of the mirror. The image size is also the same as the object size.



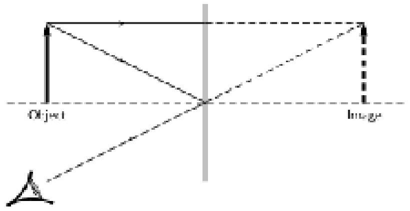
4. Place a dot at the point the eye is located.
5. Pick one point on the image and draw the reflected ray that travels to the eye as it sees this point. Remember to add an arrowhead.



6. Draw the incident ray for light traveling from the corresponding point on the object to the mirror, such that the law of reflection is obeyed.



7. Continue for other extreme points on the object (i.e. the tip and base of the arrow).



Suppose a light ray leaves the top of the object traveling parallel to the principal axis. The ray will hit the mirror at an angle of incidence of 0 degrees. We say that the ray hits the mirror *normally*. According to the law of reflection, the ray will be reflected at 0 degrees. The ray then bounces back in the same direction. We also project the ray back behind the mirror because this is what your eye does.

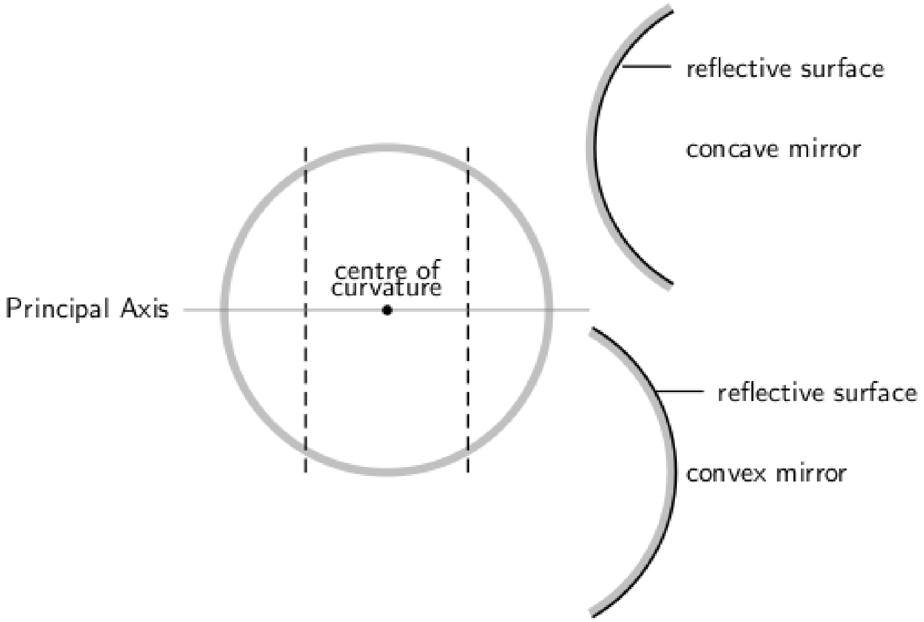
Another light ray leaves the top of the object and hits the mirror at its centre. This ray will be reflected at the same angle as its angle of incidence, as shown. If we project the ray backward behind the mirror, it will eventually cross the projection of the first ray we drew. We have found the location of the image! It is a virtual image since it appears in an area that light cannot actually reach (behind the mirror). You can see from the diagram that the image is erect and is the same size as the object. This is exactly as we expected.

We use a dashed line to indicate that the image is virtual.
Khan academy video on mirrors - 1
[missing_resource: http://www.youtube.com/v/nrOg85VPQgw&rel=0]

Spherical Mirrors

The second class of mirrors that we will look at are spherical mirrors. These mirrors are called spherical mirrors because if you take a sphere and cut it as shown in [link] and then polish the inside of one and the outside of the other, you will get a *concave mirror* and *convex mirror* as shown. These two mirrors will be studied in detail.

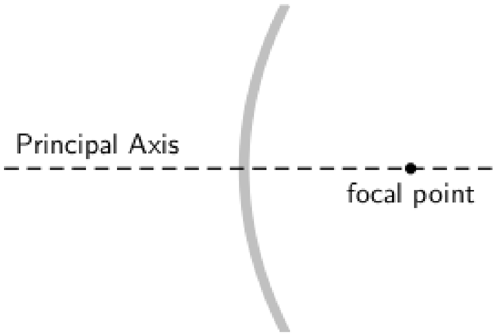
The centre of curvature is the point at the centre of the sphere and describes how big the sphere is.



When a sphere is cut and then polished to a reflective surface on the inside a concave mirror is obtained. When the outside is polished to a reflective surface, a convex mirror is obtained.

Concave Mirrors

The first type of curved mirror we will study are concave mirrors. Concave mirrors have the shape shown in [link]. As with a plane mirror, the principal axis is a line that is perpendicular to the centre of the mirror.



Concave mirror with principal axis.

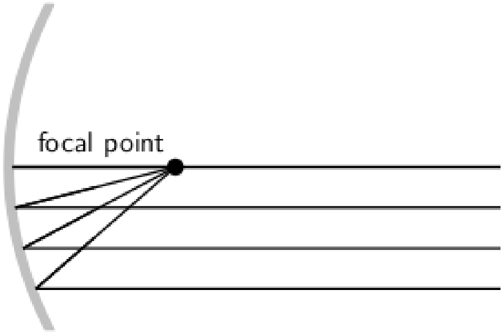
If you think of light reflecting off a concave mirror, you will immediately see that things will look very different compared to a plane mirror. The easiest way to understand what will happen is to draw a ray diagram and work out where the images will form. Once we have done that it is easy to see what properties the image has.

First we need to define a very important characteristic of the mirror. We have seen that the centre of curvature is the centre of the sphere from which the mirror is cut. We then define that a distance that is half-way between the centre of curvature and the mirror on the principal axis. This point is known as the *focal point* and the distance from the focal point to the mirror is known as the *focal length* (symbol f). Since the focal point is the midpoint of the line segment joining the vertex and the center of curvature, the focal length would be one-half the radius of curvature. This fact can come in very handy, remember if you know one then you know the other!

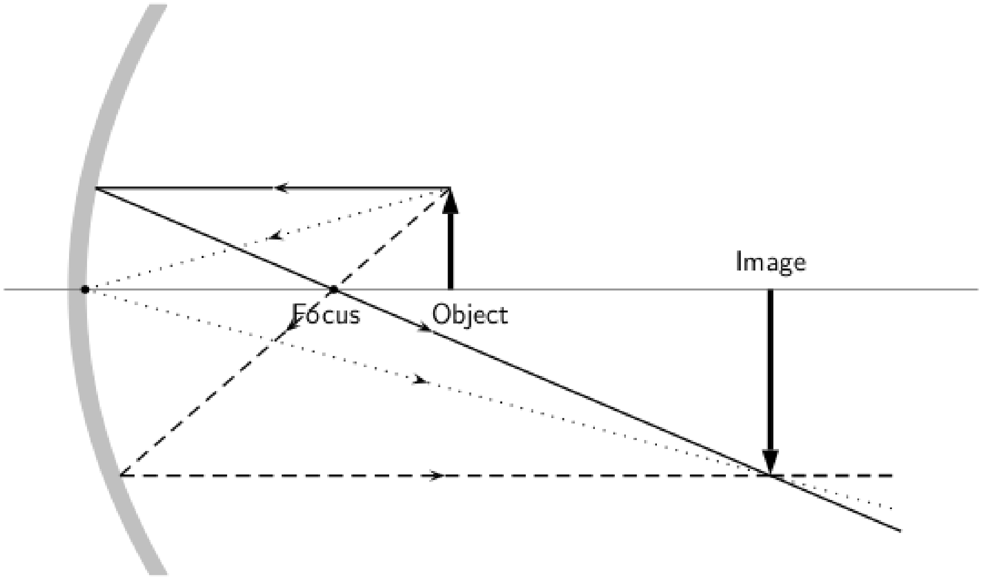
Focal Point

> The focal point of a mirror is the midpoint of a line segment joining the vertex and the centre of curvature. It is the position at which all parallel rays are focussed.

Why are we making such a big deal about this point we call the focal point? It has an important property we will use often. A ray parallel to the principal axis hitting the mirror will always be reflected through the focal point. The focal point is the position at which all parallel rays are *focussed*.



All light rays pass through the focal point.



A concave mirror with three rays drawn to locate the image. Each incident ray is reflected according to the Law of Reflection. The intersection of the reflected rays gives the location of the image. Here the image is real and inverted.

From [link], we see that the image created by a concave mirror is real and inverted, as compared to the virtual and erect image created by a plane mirror.

Real Image

> A real image can be cast on a screen; it is inverted, and on the same side of the mirror as the object.

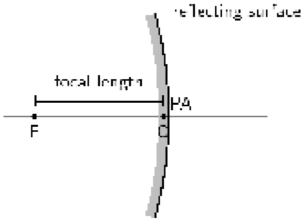
Convergence

A concave mirror is also known as a converging mirror. Light rays appear to converge to the focal point of a concave mirror.

Convex Mirrors

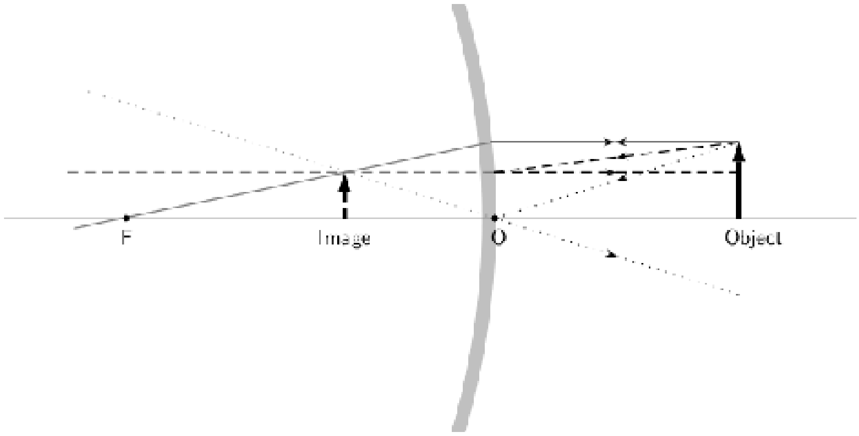
The second type of curved mirror we will study are convex mirrors. Convex mirrors have the shape shown in [link]. As with a plane mirror, the principal axis is a line that is perpendicular to the centre of the mirror.

We have defined the focal point as that point that is half-way along the principal axis between the centre of curvature and the mirror. Now for a convex mirror, this point is *behind* the mirror. A convex mirror has a *negative* focal length because the focal point is behind the mirror.



Convex mirror with principle axis, focal point (F) and centre of curvature (C). The centre of the mirror is the optical centre (O).

To determine what the image from a convex mirror looks like and where the image is located, we need to remember that a mirror obeys the laws of reflection and that light appears to come from the image. The image created by a convex mirror is shown in [link].



A convex mirror with three rays drawn to locate the image. Each incident ray is reflected according to the Law of Reflection. The reflected rays diverge. If the reflected rays are extended behind the mirror, then their intersection gives the location of the image behind the mirror. For a convex mirror, the image is virtual and upright.

From [link], we see that the image created by a convex mirror is virtual and upright, as compared to the real and inverted image created by a concave mirror.

Divergence

A convex mirror is also known as a diverging mirror. Light rays appear to diverge from the focal point of a convex mirror.

Summary of Properties of Mirrors

The properties of mirrors are summarised in [link].

Plane	**Concave**	**Convex**
–	converging	diverging
virtual image	real image	virtual image
upright	inverted	upright
image behind mirror	image in front of mirror	image behind mirror

Summary of properties of concave and convex mirrors.

Magnification

In [link] and [link], the height of the object and image arrows were different. In any optical system where images are formed from objects, the ratio of the image height, h_i, to the object height, h_o is known as the magnification, m.

Equation:

$$m = \frac{h_i}{h_o}$$

This is true for the mirror examples we showed above and will also be true for lenses, which will be introduced in the next sections. For a plane mirror, the height of the image is the same as the height of the object, so the magnification is simply $m = \frac{h_i}{h_o} = 1$. If the magnification is greater than 1, the image is larger than the object and is said to be *magnified*. If the magnification is less than 1, the image is smaller than the object so the image is said to be *diminished*.

Exercise:

Magnification

Problem:

A concave mirror forms an image that is 4,8 cm high. The height of the object is 1,6 cm. Calculate the magnification of the mirror.

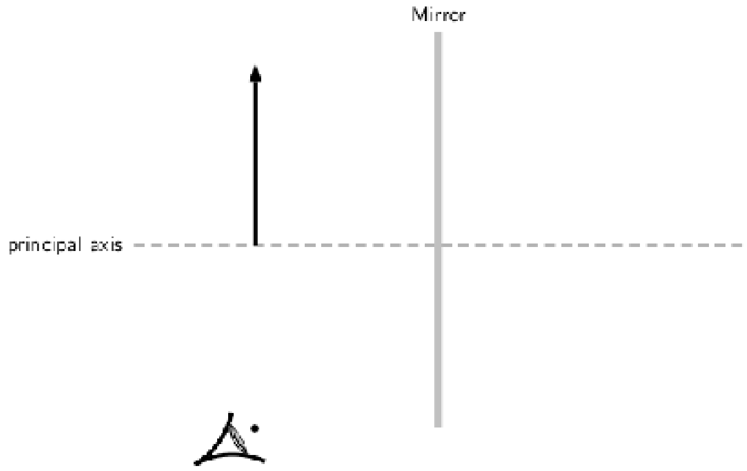
Solution:

Identify what is given and

what is asked.	Image height h_i = 4,8 cm	Object height h_o = 1,6 cm	Magnification m = ?
Substitute the values and calculate m.	**Equation:** $m = \frac{h_i}{h_o} = \frac{4,8}{1,6} = 3$		The magnification is 3 times.

Mirrors

1. List 5 properties of a virtual image created by reflection from a plane mirror.
   Click here for the solution.
2. What angle does the principal axis make with a plane mirror?
   Click here for the solution.
3. Is the principal axis a normal to the surface of the plane mirror?
   Click here for the solution.
4. Do the reflected rays that contribute to forming the image from a plane mirror obey the law of reflection?
   Click here for the solution.
5. If a candle is placed 50 cm in front of a plane mirror, how far behind the plane mirror will the image be? Draw a ray diagram to show how the image is formed.
   Click here for the solution.
6. If a stool 0,5 m high is placed 2 m in front of a plane mirror, how far behind the plane mirror will the image be and how high will the image be?
   Click here for the solution.
7. If Susan stands 3 m in front of a plane mirror, how far from Susan will her image be located?
   Click here for the solution.
8. Explain why ambulances have the word `ambulance' reversed on the front bonnet of the car?
   Click here for the solution.
9. Complete the diagram by filling in the missing lines to locate the image.



Click here for the solution.

10. An object 2 cm high is placed 4 cm in front of a plane mirror. Draw a ray diagram, showing the object, the mirror and the position of the image.
    Click here for the solution.
11. The image of an object is located 5 cm behind a plane mirror. Draw a ray diagram, showing the image, the mirror and the position of the object.
    Click here for the solution.
12. How high must a mirror be so that you can see your whole body in it? Does it make a difference if you change the distance you stand in front of the mirror? Explain.
    Click here for the solution.
13. If 1-year old Tommy crawls towards a mirror at a rate of 0,3 $m \cdot s^{-1}$, at what speed will Tommy and his image approach each other?
    Click here for the solution.
14. Use a diagram to explain how light converges to the focal point of a concave mirror.
    Click here for the solution.
15. Use a diagram to explain how light diverges away from the focal point of a convex mirror.
    Click here for the solution.
16. An object 1 cm high is placed 4 cm from a concave mirror. If the focal length of the mirror is 2 cm, find the position and size of the image by means of a ray diagram. Is the image real or virtual? What is the magnification?
    Click here for the solution.
17. An object 2 cm high is placed 4 cm from a convex mirror. If the focal length of the mirror is 4 cm, find the position and size of the image by means of a ray diagram. Is the image real or virtual? What is the magnification?
    Click here for the solution.

Total Internal Reflection and Fibre Optics

Total Internal Reflection

Investigation : Total Internal Reflection

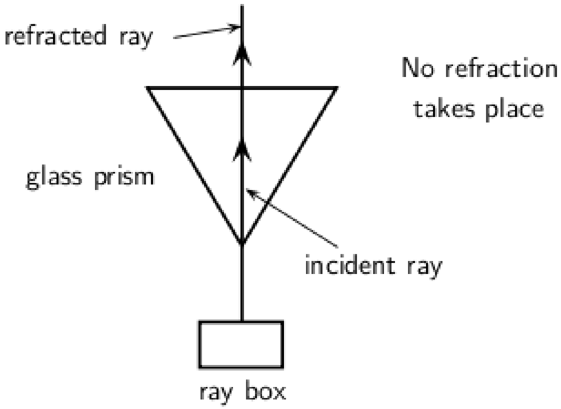
Work in groups of four. Each group will need a raybox (or torch) with slit, triangular glass prism and protractor. If you do not have a raybox, use a torch and stick two pieces of tape over the lens so that only a thin beam of light is visible.

Aim:

To investigate total internal reflection.

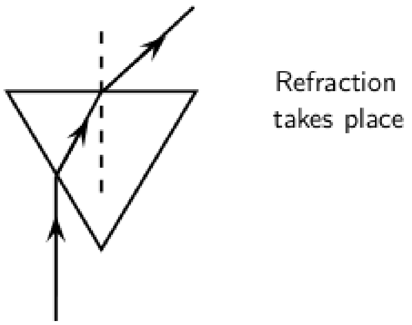
Method:

1. Place the raybox next to the glass block so that the light shines right through without any refraction. See "Position 1" in diagram.



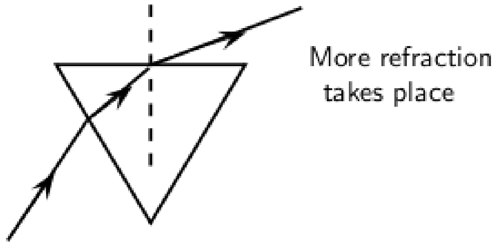
Position 1

2. Move the raybox such that the light is refracted by the glass. See "Position 2".



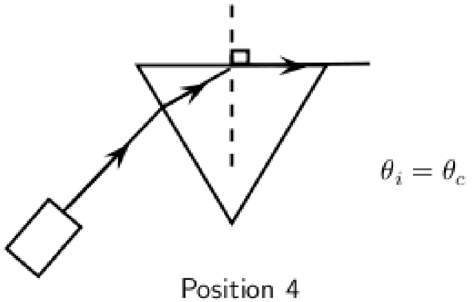
Position 2

3. Move the raybox further and observe what happens.



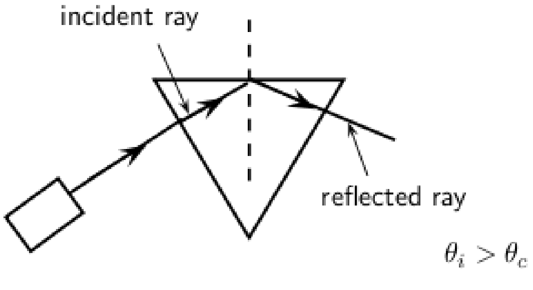
Position 3

4. Move the raybox until the refracted ray seems to disappear. See "Position 4". The angle of the incident light is called the critical angle.



Position 4

5. Move the raybox further and observe what happens. See "Position 5". The light shines back into the glass block. This is called total internal reflection.



Position 5

When we increase the angle of incidence, we reach a point where the angle of refraction is 90° and the refracted ray runs along the surface of the medium. This angle of incidence is called the critical angle.

Critical Angle

> The critical angle is the angle of incidence where the angle of reflection is 90°. The light must shine from a dense to a less dense medium.

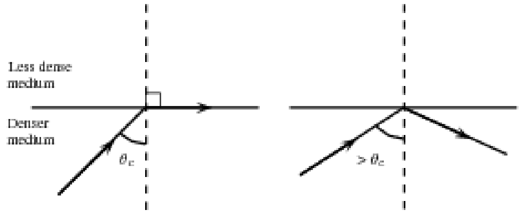
If the angle of incidence is bigger than this critical angle, the refracted ray will not emerge from the medium, but will be reflected back into the medium. This is called total internal reflection.

Total internal reflection takes place when

- light shines from an optically denser medium to an optically less dense medium.
- the angle of incidence is greater than the critical angle.

Total Internal Reflection

> Total internal reflection takes place when light is reflected back into the medium because the angle of incidence is greater than the critical angle.



Diagrams to show the critical angle and total internal reflection.

Each medium has its own unique critical angle. For example, the critical angle for glass is 42°, and that of water is 48,8°. We can calculate the critical angle for any medium.

Calculating the Critical Angle

Now we shall learn how to derive the value of the critical angle for two given media. The process is fairly simple and involves just the use of Snell's Law that we have already studied. To recap, Snell's Law states:

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

where n_1 is the refractive index of material 1, n_2 is the refractive index of material 2, θ_1 is the angle of incidence and θ_2 is the angle of refraction. For total internal reflection we know that the angle of incidence is the critical angle. So,

Equation:

$$\theta_1 = \theta_c.$$

However, we also know that the angle of refraction at the critical angle is 90°. So we have:

Equation:

$$\theta_2 = 90^\circ.$$

We can then write Snell's Law as:

Equation:

$$n_1 \sin \theta_c = n_2 \sin 90^\circ$$

Solving for θ_c gives:

Equation:

$$\begin{aligned} n_1 \sin \theta_c &= n_2 \sin 90^\circ \\ \sin \theta_c &= \frac{n_2}{n_1}(1) \\ \therefore \theta_c &= \sin^{-1}\left(\frac{n_2}{n_1}\right) \end{aligned}$$

Note: Take care that for total internal reflection the incident ray is always in the denser medium.

Khan academy video on refraction - 1
[missing_resource: http://www.youtube.com/v/WRuatAcd2WY&rel=0]
Exercise:
Critical Angle 1

Problem:

Given that the refractive indices of air and water are 1 and 1,33, respectively, find the critical angle.

Solution:

Determine how to approach the problem

We know that the critical angle is given by:

Equation:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Solve the problem

Equation:

$$\begin{aligned} \theta_c &= \sin^{-1}\left(\frac{n_2}{n_1}\right) \\ &= \sin^{-1}\left(\frac{1}{1,33}\right) \\ &= 48,8^\circ \end{aligned}$$

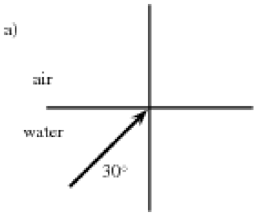
Write the final answer

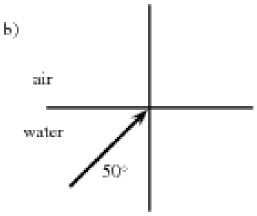
The critical angle for light travelling from water to air is $48,8^\circ$.

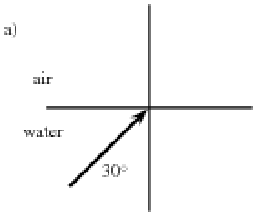
Exercise:
Critical Angle 2

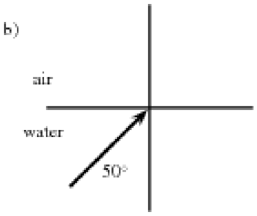
Problem:

Complete the following ray diagrams to show the path of light in each situation.

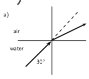
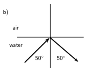
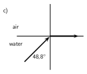
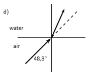








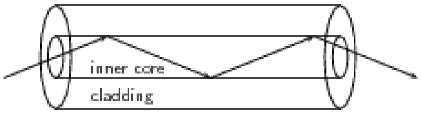
Solution:

Identify what is given and what is asked	The critical angle for water is $48,8^\circ$.	We are asked to complete the diagrams.	For incident angles smaller than $48,8^\circ$ refraction will occur.	For incident angles greater than $48,8^\circ$ total internal reflection will occur.	For incident angles equal to $48,8^\circ$ refraction will occur at 90°.	The light must travel from a high optical density to a lower one.		
Complete the diagrams	 a) air water 30°	Refraction occurs (ray is bent away from the normal)	 b) air water 50° 50°	Total internal reflection occurs	 c) air water 48,8°	$\theta_c = 48,8^\circ$	 d) water air 48,8°	Refraction towards the normal (air is less dense than water)

Fibre Optics

Total internal reflection is a powerful tool since it can be used to confine light. One of the most common applications of total internal reflection is in *fibre optics*. An optical fibre is a thin, transparent fibre, usually made of glass or plastic, for transmitting light. Optical fibres are usually thinner than a human hair! The construction of a single optical fibre is shown in [link].

The basic functional structure of an optical fibre consists of an outer protective *cladding* and an *inner core* through which light pulses travel. The overall diameter of the fibre is about 125 μm ($125 \times 10^{-6}\,\mathrm{m}$) and that of the core is just about 50 μm ($10 \times 10^{-6}\,\mathrm{m}$). The mode of operation of the optical fibres, as mentioned above, depends on the phenomenon of total internal reflection. The difference in refractive index of the cladding and the core allows total internal reflection in the same way as happens at an air-water surface. If light is incident on a cable end with an angle of incidence greater than the critical angle then the light will remain trapped inside the glass strand. In this way, light travels very quickly down the length of the cable.



Structure of a single optical fibre.

Fibre Optics in Telecommunications

Optical fibres are most common in telecommunications, because information can be transported over long distances, with minimal loss of data. The minimised loss of data gives optical fibres an advantage over conventional cables.

Data is transmitted from one end of the fibre to another in the form of laser pulses. A single strand is capable of handling over 3000 simultaneous transmissions which is a huge improvement over the conventional co-axial cables. Multiple signal transmission is achieved by sending individual light pulses at slightly different angles. For example if one of the pulses makes a 72,23° angle of incidence then a separate pulse can be sent at an angle of 72,26°! The transmitted data is received almost instantaneously at the other end of the cable since the information coded onto the laser travels at the speed of light! During transmission over long distances *repeater stations* are used to amplify the signal which has weakened somewhat by the time it reaches the station. The amplified signals are then relayed towards their destination and may encounter several other repeater stations on the way.

Fibre Optics in Medicine

Optic fibres are used in medicine in *endoscopes*.

> **Note:**
> Interesting Fact
> Endoscopy means *to look inside* and refers to looking inside the human body for diagnosing medical conditions.

The main part of an endoscope is the optical fibre. Light is shone down the optical fibre and a medical doctor can use the endoscope to look inside a patient. Endoscopes are used to examine the inside of a patient's stomach, by inserting the endoscope down the patient's throat.

Endoscopes allow minimally invasive surgery. This means that a person can be diagnosed and treated through a small incision. This has advantages over open surgery because endoscopy is quicker and cheaper and the patient recovers more quickly. The alternative is open surgery which is expensive, requires more time and is more traumatic for the patient.

Total Internal Reflection and Fibre Optics

1. Describe total internal reflection, referring to the conditions that must be satisfied for total internal reflection to occur.
   Click here for the solution.
2. Define what is meant by the *critical angle* when referring to total internal reflection. Include a ray diagram to explain the concept.
   Click here for the solution.

3. Will light travelling from diamond to silicon ever undergo total internal reflection?
   Click here for the solution.
4. Will light travelling from sapphire to diamond undergo total internal reflection?
   Click here for the solution.
5. What is the critical angle for light traveling from air to acetone?
   Click here for the solution.
6. Light traveling from diamond to water strikes the interface with an angle of incidence of 86°. Calculate the critical angle to determine whether the light be totally internally reflected and so be trapped within the water.
   Click here for the solution.
7. Which of the following interfaces will have the largest critical angle?

   a. a glass to water interface
   b. a diamond to water interface
   c. a diamond to glass interface

   Click here for the solution.
8. If the fibre optic strand is made from glass, determine the critical angle of the light ray so that the ray stays within the fibre optic strand.
   Click here for the solution.
9. A glass slab is inserted in a tank of water. If the refractive index of water is 1,33 and that of glass is 1,5, find the critical angle.
   Click here for the solution.
10. A diamond ring is placed in a container full of glycerin. If the critical angle is found to be 37,4° and the refractive index of glycerin is given to be 1,47, find the refractive index of diamond.
    Click here for the solution.
11. An optical fibre is made up of a core of refractive index 1,9, while the refractive index of the cladding is 1,5. Calculate the maximum angle which a light pulse can make with the wall of the core. NOTE: The question does not ask for the angle of incidence but for the angle made by the ray with the wall of the core, which will be equal to 90°- angle of incidence.
    Click here for the solution.

[missing_resource: http://static.slidesharecdn.com/swf/ssplayer2.swf?doc=lightg10-100511072123-phpapp01&stripped_title=light-g10&userName=kwarne]

Summary

1. We can see objects when light from the objects enters our eyes.
2. Light rays are thin imaginary lines of light and are indicated in drawings by means of arrows.
3. Light travels in straight lines. Light can therefore not travel around corners. Shadows are formed because light shines in straight lines.
4. Light rays reflect off surfaces. The incident ray shines in on the surface and the reflected ray is the one that bounces off the surface. The surface normal is the perpendicular line to the surface where the light strikes the surface.
5. The angle of incidence is the angle between the incident ray and the surface, and the angle of reflection is the angle between the reflected ray and the surface.
6. The Law of Reflection states the angle of incidence is equal to the angle of reflection and that the reflected ray lies in the plane of incidence.
7. Specular reflection takes place when parallel rays fall on a surface and they leave the object as parallel rays. Diffuse reflection takes place when parallel rays are reflected in different directions.
8. Refraction is the bending of light when it travels from one medium to another. Light travels at different speeds in different media.
9. The refractive index of a medium is a measure of how easily light travels through the medium. It is a ratio of the speed of light in a vacuum to the speed of light in the medium. $n = \frac{c}{v}$
10. Snell's Law gives the relationship between the refractive indices, angles of incidence and reflection of two media. $n_1 \sin\theta_1 = n_2 \sin\theta_2$
11. Light travelling from one medium to another of lighter optical density will be refracted towards the normal. Light travelling from one medium to another of lower optical density will be refracted away from the normal.
12. Objects in a medium (e.g. under water) appear closer to the surface than they really are. This is due to the refraction of light, and the refractive index of the medium. $n = \frac{\text{real depth}}{\text{apparent depth}}$
13. Mirrors are highly reflective surfaces. Flat mirrors are called plane mirrors. Curved mirrors can be convex or concave. The properties of the images formed by mirrors are summarised in Table 3.2.
14. A real image can be cast on a screen, is inverted and in front of the mirror. A virtual image cannot be cast on a screen, is upright and behind the mirror.
15. The magnification of a mirror is how many times the image is bigger or smaller than the object. $m = \frac{\text{image height}(\text{h}_\text{i})}{\text{object height}(\text{h}_0)}$
16. The critical angle of a medium is the angle of incidence when the angle of refraction is 90° and the refracted ray runs along the interface between the two media.

17. Total internal reflection takes place when light travels from one medium to another of lower optical density. If the angle of incidence is greater than the critical angle for the medium, the light will be reflected back into the medium. No refraction takes place.
18. Total internal reflection is used in optical fibres in telecommunication and in medicine in endoscopes. Optical fibres transmit information much more quickly and accurately than traditional methods.

Exercises

1. Give one word for each of the following descriptions:

    a. The image that is formed by a plane mirror.
    b. The perpendicular line that is drawn at right angles to a reflecting surface at the point of incidence.
    c. The bending of light as it travels from one medium to another.
    d. The ray of light that falls in on an object.
    e. A type of mirror that focuses all rays behind the mirror.

    Click here for the solution.
2. State whether the following statements are TRUE or FALSE. If they are false, rewrite the statement correcting it.

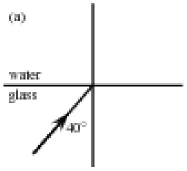
    a. The refractive index of a medium is an indication of how fast light will travel through the medium.
    b. Total internal refraction takes place when the incident angle is larger than the critical angle.
    c. The magnification of an object can be calculated if the speed of light in a vacuum and the speed of light in the medium is known.
    d. The speed of light in a vacuum is about 3×10^{8} m.s^{-1}.
    e. Specular reflection takes place when light is reflected off a rough surface.

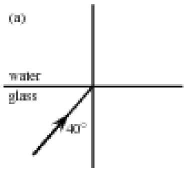
    Click here for the solution.
3. Choose words from Column B to match the concept/description in Column A. All the appropriate words should be identified. Words can be used more than once.

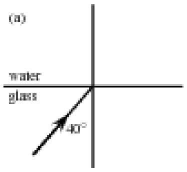
	Column A	Column B
(a)	Real image	Upright
(b)	Virtual image	Can be cast on a screen
(c)	Concave mirror	In front
(d)	Convex mirror	Behind
(e)	Plane mirror	Inverted
		Light travels to it
		Upside down
		Light does not reach it
		Erect
		Same size

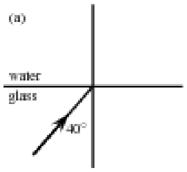
Click here for the solution.

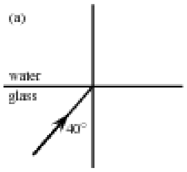
4. Complete the following ray diagrams to show the path of light.

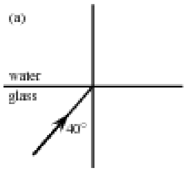












Click here for the solution.

5. A ray of light strikes a surface at 35° to the surface normal. Draw a ray diagram showing the incident ray, reflected ray and surface normal. Calculate the angles of incidence and reflection and fill them in on your diagram.
   Click here for the solution.
6. Light travels from glass (n = 1,5) to acetone (n = 1,36). The angle of incidence is 25°.

   a. Describe the path of light as it moves into the acetone.
   b. Calculate the angle of refraction.
   c. What happens to the speed of the light as it moves from the glass to the acetone?
   d. What happens to the wavelength of the light as it moves into the acetone?
   e. What is the name of the phenomenon that occurs at the interface between the two media?

   Click here for the solution.
7. A stone lies at the bottom of a swimming pool. The water is 120 cm deep. The refractive index of water is 1,33. How deep does the stone appear to be?
   Click here for the solution.
8. Light strikes the interface between air and an unknown medium with an incident angle of 32°. The angle of refraction is measured to be 48°. Calculate the refractive index of the medium and identify the medium.
   Click here for the solution.
9. Explain what total internal reflection is and how it is used in medicine and telecommunications. Why is this technology much better to use?
   Click here for the solution.

10. A candle 10 cm high is placed 25 cm in front of a plane mirror. Draw a ray diagram to show how the image is formed. Include all labels and write down the properties of the image.
    Click here for the solution.
11. A virtual image, 4 cm high, is formed 3 cm from a plane mirror. Draw a labelled ray diagram to show the position and height of the object. What is the magnification?
    Click here for the solution.
12. An object, 3 cm high, is placed 4 cm from a concave mirror of focal length 2 cm. Draw a labelled ray diagram to find the position, height and properties of the image.
    Click here for the solution.

Force, Momentum and Impulse

Introduction

In Grade 10 we studied motion but not what caused the motion. In this chapter we will learn that a net force is needed to cause motion. We recall what a force is and learn about how force and motion are related. We are introduced to two new concepts, momentum and impulse, and we learn more about turning forces and the force of gravity.

Force

What is a *force*?

A force is anything that can cause a change to objects. Forces can:

- change the shape of an object
- accelerate or stop an object
- change the direction of a moving object.

A force can be classified as either a *contact force* or a *non-contact force*.

A contact force must touch or *be in contact* with an object to cause a change. Examples of contact forces are:

- the force that is used to push or pull things, like on a door to open or close it
- the force that a sculptor uses to turn clay into a pot
- the force of the wind to turn a windmill

Write down 5 examples (excluding those given above) of contact forces that you see on your way to school.

A non-contact force does not have to touch an object to cause a change. Examples of non-contact forces are:

- the force due to gravity, like the Earth pulling the Moon towards itself
- the force due to electricity, like a proton and an electron attracting each other
- the force due to magnetism, like a magnet pulling a paper clip towards itself

The unit of force is the *newton* (symbol *N*). This unit is named after Sir Isaac Newton who first defined force. Force is a vector quantity and has a magnitude and a direction. We use the abbreviation *F* for force.

Note:
Interesting Fact
There is a popular story that while Sir Isaac Newton was sitting under an apple tree, an apple fell on his head, and he suddenly thought of the Universal Law of Gravitation. Coincidently, the weight of a small apple is approximately 1 N.

Note:
Interesting Fact
Force was first described by Archimedes of Syracuse (circa 287 BC - 212 BC). Archimedes was a Greek mathematician, astronomer, philosopher, physicist and engineer. He was killed by a Roman soldier during the sack of the city, despite orders from the Roman general, Marcellus, that he was not to be harmed.

This chapter will often refer to the *resultant force* acting on an object. The resultant force is simply the vector sum of all the forces acting on the object. It is very important to remember that all the forces must be acting on the *same* object. The resultant force is the force that has the same effect as all the other forces added together.

Examples of Forces in Physics

Most of Physics revolves around the study of forces. Although there are many different forces, all are handled in the same way. All forces in Physics can be put into one of four groups. These are gravitational forces, electromagnetic forces, strong nuclear forces and weak nuclear forces. You will mostly come across gravitational or electromagnetic forces at school.

Gravitational Forces

Gravity is the attractive force between two objects due to the mass of the objects. When you throw a ball in the air, its mass and the Earth's mass attract each other, which leads to a force between them. The ball falls back towards the Earth, and the Earth accelerates towards the ball. The movement of the Earth towards the ball is, however, so small that you couldn't possibly measure it.

Electromagnetic Forces

Almost all of the forces that we experience in everyday life are electromagnetic in origin. They have this unusual name because long ago people thought that electric forces and magnetic forces were different things. After much work and experimentation, it has been realised that they are actually different manifestations of the same underlying theory.

Electric or Electrostatic Forces

If we have objects carrying electrical charge, which are not moving, then we are dealing with electrostatic forces (Coulomb's Law). This force is actually much stronger than gravity. This may seem strange, since gravity is obviously very powerful, and holding a balloon to the wall seems to be the most impressive thing electrostatic forces have done, but if we think about it: for gravity to be detectable, we need to have a very large mass nearby. But a balloon rubbed against someone's hair can stick to a wall with a force so strong that it overcomes the force of gravity between the entire Earth and the balloon—with just the charges in the balloon and the wall!

Magnetic Forces

The magnetic force is a different manifestation of the electromagnetic force. It stems from the interaction between *moving charges* as opposed to the *fixed* charges involved in Coulomb's Law. Examples of the magnetic force in action include magnets, compasses, car engines and computer data storage. Magnets are also used in the wrecking industry to pick up cars and move them around sites.

Friction

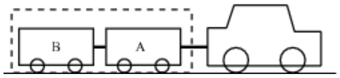
According to Newton's First Law (we will discuss this later in the chapter) an object moving *without a force acting on it* will keep on moving. Then why does a box sliding on a table come to a stop? The answer is friction. Friction arises where two surfaces are in contact and moving relative to eachother as a result of the interaction between the molecules of the two contact surfaces—for instance the interactions between the molecules on the bottom of the box with molecules on the top of the table. This interaction is electromagnetic in origin, hence friction is just another view of the electromagnetic force. Later in this chapter we will discuss frictional forces a little more.

Drag Forces

This is the force an object experiences while travelling through a medium like an aeroplane flying through air. When something travels through the air it needs to displace air as it travels and because of this, the air exerts a force on the object. This becomes an important force when you move fast and a lot of thought is taken to try and reduce the amount of drag force a sports car or an aeroplane experiences. The drag force is very useful for parachutists. They jump from high altitudes and if there was no drag force, then they would continue accelerating all the way to the ground. Parachutes are wide because the more surface area you have, the greater the drag force and hence the slower you hit the ground.

Systems and External Forces

The concepts of systems and forces external to such systems are very important in Physics. A system is any collection of objects. If one draws an imaginary box around such a system then an external force is one that is applied by an object or person outside the box. Imagine for example a car pulling two trailers.



If we draw a box around the two trailers they can be considered a closed system or unit. When we look at the forces on this closed system the following forces will apply (we assume drag forces are absent):

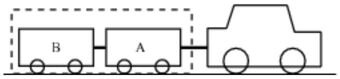
- The force of the car pulling the unit (trailer A and B)
- The force of friction between the wheels of the trailers and the road (opposite to the direction of motion)
- The force of the Earth pulling downwards on the system (gravity)
- The force of the road pushing upwards on the system

These forces are called external forces to the system.

The following forces will not apply:

- The force of A pulling B
- The force of B pulling A
- The force of friction between the wheels of the car and the road (opposite to the direction of motion)

We can also draw a box around trailer A or B, in which case the forces will be different.

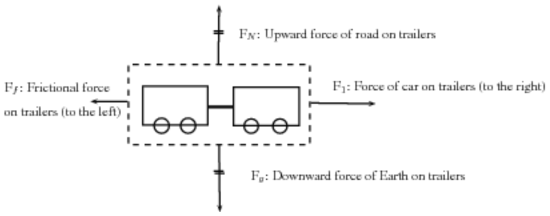


If we consider trailer A as a system, the following external forces will apply:

- The force of the car pulling on A (towards the right)
- The force of B pulling on A (towards the left)
- The force of the Earth pulling downwards on the trailer (gravity)
- The force of the road pushing upwards on the trailer
- The force of friction between the wheels of A and the road (opposite to the direction of motion)

Force Diagrams

If we look at the example above and draw a force diagram of all the forces acting on the two-trailer-unit, the diagram would look like this:


F_N: Upward force of road on trailers
F_f: Frictional force on trailers (to the left)
F_1: Force of car on trailers (to the right)
F_g: Downward force of Earth on trailers

It is important to keep the following in mind when you draw force diagrams:

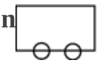
- Make your drawing large and clear.
- You must use arrows and the direction of the arrow will show the direction of the force.
- The length of the arrow will indicate the size of the force, in other words, the longer arrows in the diagram (F_1 for example) indicates a bigger force than a shorter arrow (F_f). Arrows of the same length indicate forces of equal size (F_N and F_g). Use “little lines” like in maths to show this.
- Draw neat lines using a ruler. The arrows must touch the system or object.
- All arrows must have labels. Use letters with a key on the side if you do not have enough space on your drawing.
- The labels must indicate what is applying the force (the force of the car?) on what the force is applied (?on the trailer?) and in which direction (to the right)
- If the values of the forces are known, these values can be added to the diagram or key.

Exercise:
Force diagrams

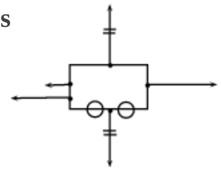
Problem: Draw a labeled force diagram to indicate all the forces acting on trailer A in the example above.

Solution:

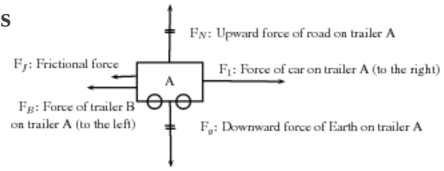
Draw a large diagram of the ?picture? from your question



Add all the forces

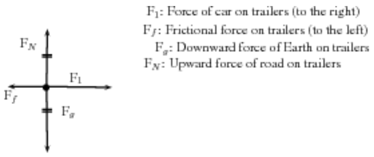


Add the labels


F_N: Upward force of road on trailer A
F_f: Frictional force
A
F_1: Force of car on trailer A (to the right)
F_B: Force of trailer B on trailer A (to the left)
F_g: Downward force of Earth on trailer A

Free Body Diagrams

In a free-body diagram, the object of interest is drawn as a dot and all the forces acting on it are drawn as arrows pointing away from the dot. A free body diagram for the two-trailer-system will therefore look like this:

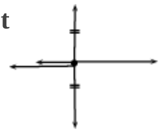

F_N
F_1
F_f
F_g
F_1: Force of car on trailers (to the right)
F_f: Frictional force on trailers (to the left)
F_g: Downward force of Earth on trailers
F_N: Upward force of road on trailers

Exercise:
Free body diagram

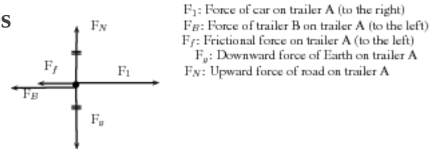
Problem: Draw a free body diagram of all the forces acting on trailer A in the example above.

Solution:

Draw a dot to indicate the object.
Draw arrows to indicate all the forces acting on the object



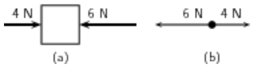
Label the forces


F_N
F_f
F_1
F_B
F_g
F_1: Force of car on trailer A (to the right)
F_B: Force of trailer B on trailer A (to the left)
F_f: Frictional force on trailer A (to the left)
F_g: Downward force of Earth on trailer A
F_N: Upward force of road on trailer A

Finding the Resultant Force

The easiest way to determine a resultant force is to draw a free body diagram. Remember from Chapter [link] that we use the length of the arrow to indicate the vector's magnitude and the direction of the arrow to show which direction it acts in.

After we have done this, we have a diagram of vectors and we simply find the sum of the vectors to get the resultant force.



(a) Force diagram of 2 forces acting on a box. (b) Free body diagram of the box.

For example, two people push on a box from opposite sides with forces of 4 N and 6 N respectively as shown in [link](a). The free body diagram in [link](b) shows the object represented by a dot and the two forces are represented by arrows with their tails on the dot.

As you can see, the arrows point in opposite directions and have different lengths. The resultant force is 2 N to the left. This result can be obtained algebraically too, since the two forces act along the same line. First, as in motion in one direction, choose a frame of reference. Secondly, add the two vectors taking their directions into account.

For the example, assume that the positive direction is to the right, then:

Equation:

$$\begin{aligned} F_R &= (+4\,\mathrm{N}) + (-6\,\mathrm{N}) \\ &= -2\,\mathrm{N} \\ &= 2\,\mathrm{N}\ \ \text{to the left} \end{aligned}$$

Remember that a negative answer means that the force acts in the *opposite* direction to the one that you chose to be positive. You can *choose* the positive direction to be any way you want, but once you have chosen it you *must* keep it.

As you work with more force diagrams in which the forces exactly balance, you may notice that you get a zero answer (e.g. 0 N). This simply means that the forces are balanced and that the object will not accelerate.

Once a force diagram has been drawn the techniques of vector addition introduced in Chapter [link] can be used. Depending on the situation you might choose to use a graphical technique such as the tail-to-head method or the parallelogram method, or else an algebraic approach to determine the resultant. Since force is a vector quantity all of these methods apply.

Exercise:

Finding the resultant force

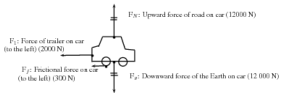
Problem:

A car (mass 1200 kg) applies a force of 2000 N on a trailer (mass 250 kg). A constant frictional force of 200 N is acting on the trailer, and a constant frictional force of 300 N is acting on the car.

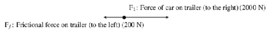
1. Draw a force diagram of all the forces acting on the car.
2. Draw a free body diagram of all the horizontal forces acting on the trailer.
3. Use the force diagram to determine the resultant force on the trailer.

Solution:

Draw the force diagram for the car. The question asks us to draw all the forces on the car. This means that we must include horizontal and vertical forces.

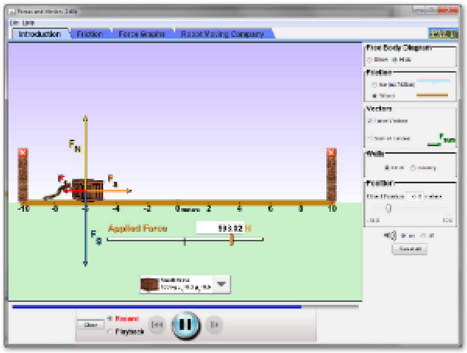

F_N: Upward force of road on car (12000 N)
F_1: Force of trailer on car (to the left) (2000 N)
F_f: Frictional force on car (to the left) (300 N)
F_g: Downward force of the Earth on car (12 000 N)

Draw the free body diagram for the trailer. The question only asks for horizontal forces. We will therefore not include the force of the Earth on the trailer, or the force of the road on the trailer as these forces are in a vertical direction.


F_1: Force of car on trailer (to the right) (2000 N)
F_f: Frictional force on trailer (to the left) (200 N)

Determine the resultant force on the trailer. To find the resultant force we need to add all the horizontal forces together. We do not add vertical forces as the movement of the car and trailer will be in a horizontal direction, and not up or down. $F_R = 2000 + (-200) = 1800$ N to the right.

run demo



Exercise

1. A force acts on an object. Name three effects that the force can have on the object.
2. Identify each of the following forces as contact or non-contact forces.
    a. The force between the north pole of a magnet and a paper clip.
    b. The force required to open the door of a taxi.
    c. The force required to stop a soccer ball.
    d. The force causing a ball, dropped from a height of 2 m, to fall to the floor.
3. A book of mass 2 kg is lying on a table. Draw a labeled force diagram indicating all the forces on the book.
4. A boy pushes a shopping trolley (mass 15 kg) with a constant force of 75 N. A constant frictional force of 20 N is present.
    a. Draw a labeled force diagram to identify all the forces acting on the shopping trolley.
    b. Draw a free body diagram of all the horizontal forces acting on the trolley.
    c. Determine the resultant force on the trolley.
5. A donkey (mass 250 kg) is trying to pull a cart (mass 80 kg) with a force of 400 N. The rope between the donkey and the cart makes an angle of 30° with the cart. The cart does not move.
    a. Draw a free body diagram of all the forces acting on the donkey.
    b. Draw a force diagram of all the forces acting on the cart.
    c. Find the magnitude and direction of the frictional force preventing the cart from moving.

Newton's Laws

In grade 10 you learned about motion, but did not look at how things start to move. You have also learned about forces. In this section we will look at the effect of forces on objects and how we can make things move.

Newton's First Law

Sir Isaac Newton was a scientist who lived in England (1642-1727) who was interested in the motion of objects under various conditions. He suggested that a stationary object will remain stationary unless a force acts on it and that a moving object will continue moving unless a force slows it down, speeds it up or changes its direction of motion. From this he formulated what is known as Newton's First Law of Motion:

Newton's First Law of Motion

> An object will remain in a state of rest or continue traveling at constant velocity, unless acted upon by an unbalanced (net) force.

Let us consider the following situations:

An ice skater pushes herself away from the side of the ice rink and skates across the ice. She will continue to move in a straight line across the ice unless something stops her. Objects are also like that. If we kick a soccer ball across a soccer field, according to Newton's First Law, the soccer ball should keep on moving forever! However, in real life this does not happen. Is Newton's Law wrong? Not really. Newton's First Law applies to situations where there aren't any external forces present. This means that friction is not present. In the case of the ice skater, the friction between the skates and the ice is very little and she will continue moving for quite a distance. In the case of the soccer ball, air resistance (friction between the air and the ball) and friction between the grass and the ball is present and this will slow the ball down.

Khan academy video on newtons laws - 1

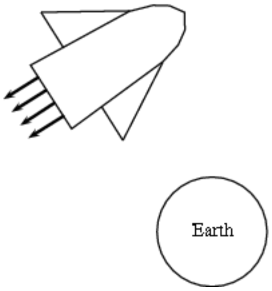
[missing_resource: http://www.youtube.com/v/CQYELiTtUs8&rel=0&hl=en_US&feature=player_embedded&version=3]

Newton's First Law in action

We experience Newton's First Law in every day life. Let us look at the following examples:

Rockets:

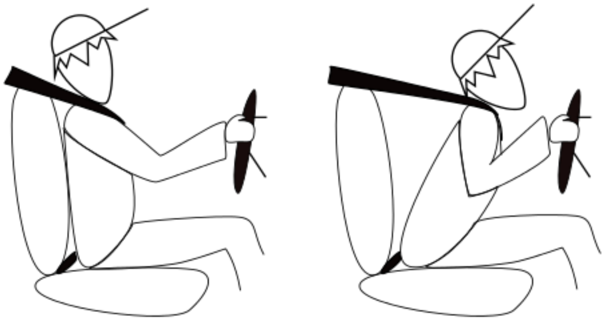
A spaceship is launched into space. The force of the exploding gases pushes the rocket through the air into space. Once it is in space, the engines are switched off and it will keep on moving at a constant velocity. If the astronauts want to change the direction of the spaceship they need to fire an engine. This will then apply a force on the rocket and it will change its direction.



Newton's First Law and rockets

Seat belts:

We wear seat belts in cars. This is to protect us when the car is involved in an accident. If a car is traveling at 120 km·hr^{-1}, the passengers in the car is also traveling at 120 km·hr^{-1}. When the car suddenly stops a force is exerted on the car (making it slow down), but not on the passengers. The passengers will carry on moving forward at 120 km·hr^{-1}according to Newton I. If they are wearing seat belts, the seat belts will stop them by exerting a force on them and so prevent them from getting hurt.

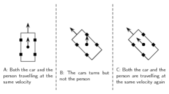


Exercise:
Newton's First Law in action

Problem: Why do passengers get thrown to the side when the car they are driving in goes around a corner?

Solution:

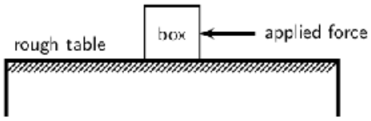
What happens before the car turns	Before the car starts turning both the passengers and the car are traveling at the same velocity. (picture A)
What	

happens while the car turns	The driver turns the wheels of the car, which then exert a force on the car and the car turns. This force acts on the car but not the passengers, hence (by Newton's First Law) the passengers continue moving with the same original velocity. (picture B)
Why passengers get thrown to the side?	If the passengers are wearing seat belts they will exert a force on the passengers until the passengers' velocity is the same as that of the car (picture C). Without a seat belt the passenger may hit the side of the car.  A: Both the car and the person travelling at the same velocity B: The cars turns but not the person C: Both the car and the person are travelling at the same velocity again

Newton's Second Law of Motion

According to Newton I, things 'like to keep on doing what they are doing'. In other words, if an object is moving, it tends to continue moving (in a straight line and at the same speed) and if an object is stationary, it tends to remain stationary. So how do objects start moving?

Let us look at the example of a 10 kg box on a rough table. If we push lightly on the box as indicated in the diagram, the box won't move. Let's say we applied a force of 100 N, yet the box remains stationary. At this point a frictional force of 100 N is acting on the box, preventing the box from moving. If we increase the force, let's say to 150 N and the box almost starts to move, the frictional force is 150 N. To be able to move the box, we need to push hard enough to overcome the friction and then move the box. If we therefore apply a force of 200 N remembering that a frictional force of 150 N is present, the 'first' 150 N will be used to overcome or 'cancel' the friction and the other 50 N will be used to move (accelerate) the block. In order to accelerate an object we must have a resultant force acting on the block.



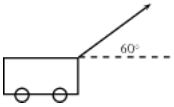
Now, what do you think will happen if we pushed harder, lets say 300 N? Or, what do you think will happen if the mass of the block was more, say 20 kg, or what if it was less? Let us investigate how the motion of an object is affected by mass and force.

Investigation : Newton's Second Law of Motion

Aim:

To investigate the relation between the acceleration of objects and the application of a constant resultant force.

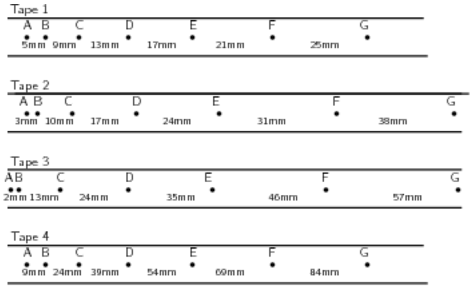
Method:



1. A constant force of 20 N, acting at an angle of 60° to the horizontal, is applied to a dynamics trolley.
2. Ticker tape attached to the trolley runs through a ticker timer of frequency 20 Hz as the trolley is moving on the frictionless surface.
3. The above procedure is repeated 4 times, each time using the same force, but varying the mass of the trolley as follows:

    - Case 1: 6,25 kg
    - Case 2: 3,57 kg
    - Case 3: 2,27 kg
    - Case 4: 1,67 kg

4. Shown below are sections of the four ticker tapes obtained. The tapes are marked with the letters A, B, C, D, etc. A is the first dot, B is the second dot and so on. The distance between each dot is also shown.


Tape 1
A B C D E F G
5mm 9mm 13mm 17mm 21mm 25mm
Tape 2
A B C D E F G
3mm 10mm 17mm 24mm 31mm 38mm
Tape 3
AB C D E F G
2mm 13mm 24mm 35mm 46mm 57mm
Tape 4
A B C D E F G
9mm 24mm 39mm 54mm 69mm 84mm

Tapes are not drawn to scale.

Instructions:

1. Use each tape to calculate the instantaneous velocity (in m·s^{-1}) of the trolley at points B and F (remember to convert the distances to m first!). Use these velocities to calculate the trolley's acceleration in each case.
2. Tabulate the mass and corresponding acceleration values as calculated in each case. Ensure that each column and row in your table is appropriately labeled.
3. Draw a graph of acceleration vs. mass, using a scale of 1 cm = 1 m·s^{-2}on the y-axis and 1 cm = 1 kg on the x-axis.
4. Use your graph to read off the acceleration of the trolley if its mass is 5 kg.
5. Write down a conclusion for the experiment.

You will have noted in the investigation above that the heavier the trolley is, the slower it moved. The acceleration is *inversely* proportional to the mass. In mathematical terms:
Equation:

$$a \propto \frac{1}{m}$$

In a similar investigation where the mass is kept constant, but the applied force is varied, you will find that the bigger the force is, the faster the object will move. The acceleration of the trolley is therefore *directly* proportional to the resultant force. In mathematical terms:
Equation:

$$a \propto F$$

Rearranging the above equations, we get $a \propto \frac{F}{m}$ OR $F = ma$

Newton formulated his second law as follows:

Newton's Second Law of Motion

> If a resultant force acts on a body, it will cause the body to accelerate in the direction of the resultant force. The acceleration of the body will be directly proportional to the resultant force and inversely proportional to the mass of the body. The mathematical representation is:

Equation:

$$F = ma.$$

Khan academy video on newtons laws - 2

[missing_resource: http://www.youtube.com/v/ou9YMWlJgkE&rel=0&hl=en_US&feature=player_embedded&version=3]

Applying Newton's Second Law

Newton's Second Law can be applied to a variety of situations. We will look at the main types of examples that you need to study.

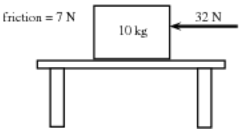
Exercise:

Newton II - Box on a surface 1

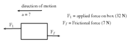
Problem:

A 10 kg box is placed on a table. A horizontal force of 32 N is applied to the box. A frictional force of 7 N is present between the surface and the box.

1. Draw a force diagram indicating all the horizontal forces acting on the box.
2. Calculate the acceleration of the box.



Solution:

Identify the horizontal forces and draw a force diagram	We only look at the forces acting in a horizontal direction (left-right) and not vertical (up-down) forces. The applied force and the force of friction will be included. The force of gravity, which is a vertical force, will not be included.	 direction of motion a = ? F_1 F_f F_1 = applied force on box (32 N) F_f = Frictional force (7 N)
Calculate the acceleration of the box	We have been given: Applied force F_1 = 32 N Frictional force F_f = - 7 N Mass m = 10 kg To calculate the acceleration of the box we will be using the equation $F_R = ma$. Therefore:	**Equation:** $F_R = ma$ $F_1 + F_f = (10)(a)$ $32 - 7 = 10\ a$ $25 = 10\ a$ $a = 2,5\,\mathrm{m \cdot s^{-1}}$ towards the left

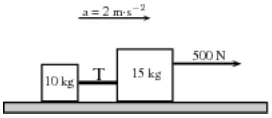
Exercise:

Newton II - box on surface 2

Problem:

Two crates, 10 kg and 15 kg respectively, are connected with a thick rope according to the diagram. A force of 500 N is applied. The boxes move with an acceleration of 2 $\mathrm{m \cdot s^{-2}}$. One third of the total frictional force is acting on the 10 kg block and two thirds on the 15 kg block. Calculate:

1. the magnitude and direction of the frictional force present.
2. the magnitude of the tension in the rope at T.

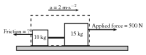


Two crates on a surface

Solution:

Draw a force diagram

Always draw a force diagram although the question might not ask for it. The acceleration of the whole system is given, therefore a force diagram of the whole system will be drawn. Because the two crates are seen as a unit, the force diagram will look like this:



Force diagram for two crates on a surface

Calculate the frictional force

To find the frictional force we will apply Newton's Second Law. We are given the mass (10 + 15 kg) and the acceleration (2 m·s^{-2}). Choose the direction of motion to be the positive direction (to the right is positive).

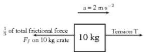
Equation:

$$F_R = ma$$
$$F_{\text{applied}} + F_f = ma$$
$$500 + F_f = (10 + 15)\,(2)$$
$$F_f = 50 - 500$$
$$F_f = -450N$$

The frictional force is 450 N opposite to the direction of motion (to the left).

Find the tension in the rope

To find the tension in the rope we need to look at one of the two crates on their own. Let's choose the 10 kg crate. Firstly, we need to draw a force diagram:



Force diagram of 10 kg crate

The frictional force on the 10 kg block is one third of the total, therefore:

$F_f = \frac{1}{3} \times F_f = 150$ N
450

If we apply Newton's Second Law:

Equation:

$$\begin{aligned} F_R &= ma \\ T + F_f &= (10)(2) \\ T + (-150) &= 20 \\ T &= 170\ \text{N} \end{aligned}$$

Note: If we had used the same principle and applied it to 15 kg crate, our calculations would have been the following:

Ec

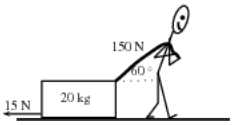
5

Exercise:

Newton II - Man pulling a box

Problem:

A man is pulling a 20 kg box with a rope that makes an angle of 60° with the horizontal. If he applies a force of 150 N and a frictional force of 15 N is present, calculate the acceleration of the box.

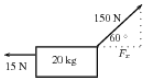


Man pulling a box

Solution:

Draw a force diagram

The motion is horizontal and therefore we will only consider the forces in a horizontal direction. Remember that vertical forces do not influence horizontal motion and vice versa.



Force diagram

Calculate the horizontal component of the applied force

The applied force is acting at an angle of 60 ° to the horizontal. We can only consider forces that are parallel to the motion. The horizontal component of the applied force needs to be calculated before we can continue:

Equation:

$$\begin{aligned} F_x &= 150 \ \cos \ 60^\circ \\ F_x &= \ 75\text{N} \end{aligned}$$

Calculate the acceleration

To find the acceleration we apply Newton's Second Law:

Equation:

$$\begin{aligned} F_R &= ma \\ F_x + F_f &= (20)(a) \\ 75 + (-15) &= 20a \\ a &= 3\,\text{m}\cdot\text{s}^{-2} \text{ to the right} \end{aligned}$$

Exercise:

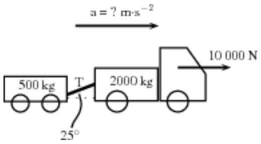
Newton II - Truck and trailor

Problem:

A 2000 kg truck pulls a 500 kg trailer with a constant acceleration. The engine of the truck produces a thrust of 10 000 N. Ignore the effect of friction.

1. Calculate the acceleration of the truck.

2. Calculate the tension in the tow bar T between the truck and the trailer, if the tow bar makes an angle of 25° with the horizontal.

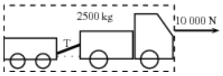


Truck pulling a trailer

Solution:

Draw a force diagram

Draw a force diagram indicating all the horizontal forces on the system as a whole:



Force diagram for truck pulling a trailer

Find the acceleration of the system

In the absence of friction, the only force that causes the system to accelerate is the thrust of the engine. If we now apply Newton's Second Law:

Equation:

$$\begin{aligned} F_R &= ma \\ 10000 &= (500+2000)a \\ a &= 4\,\mathrm{m \cdot s^{-2}} \ \text{ to the right} \end{aligned}$$

Find the horizontal component of T

We are asked to find the tension in the tow bar, but because the tow bar is acting at an angle, we need to find the horizontal component first. We will find the horizontal component in terms of T and then use it in the next step to find T.

The horizontal component is T cos 25°.

Find the tension in the tow bar

To find T, we will apply Newton's Second Law:

Equation:

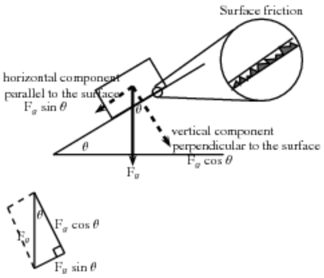
$$\begin{aligned} F_R &= ma \\ F - T\ \cos\ 25^\circ &= ma \\ 10000 - T\ \cos\ 25^\circ &= (2000)(4) \\ T\ \cos\ 25^\circ &= 2000 \\ T &= 2206,76\mathrm{N} \end{aligned}$$

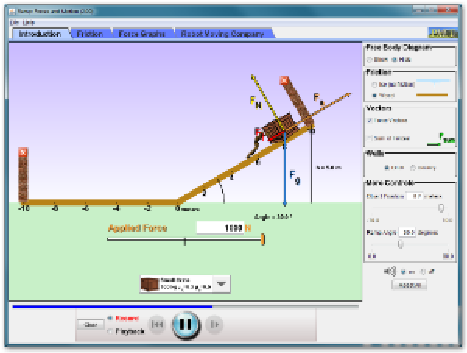
Object on an inclined plane

When we place an object on a slope the force of gravity (F_g) acts straight down and not perpendicular to the slope. Due to gravity pulling straight down, the object will tend to slide down the slope with a force equal to the horizontal component of the force of gravity ($F_g \sin \theta$). The object will 'stick' to the slope due to the frictional force between the object and the surface. As you increase the angle of the slope, the horizontal component will also increase until the frictional force is overcome and the object starts to slide down the slope.

The force of gravity will also tend to push an object 'into' the slope. The vertical component of this force is equal to the vertical component of the force of gravity ($F_g \cos\theta$). There is no movement in this direction as this force is balanced by the slope pushing up against the object. This “pushing force” is called the normal force (N) and is equal to the force required to make the component of the resultant force perpendicularly into the plane zero, $F_g \cos\theta$ in this case, but opposite in direction.

Note: Do not use the abbreviation W for weight as it is used to abbreviate 'work'. Rather use the force of gravity F_g for weight.



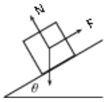


[run demo](#)

Exercise:

Newton II - Box on inclined plane

Problem: A body of mass M is at rest on an inclined plane.



What is the magnitude of the frictional force acting on the body?

A. Mg
B. Mg $\cos\theta$
C. Mg $\sin\theta$
D. Mg $\tan\theta$

Solution:

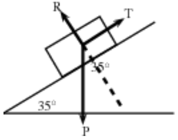
Analyse the situation The question asks us to identify the frictional force. The body is said to be at rest on the plane, which means that it is not moving and therefore there is no resultant force. The frictional force must therefore be balanced by the force F up the inclined plane.

Choose the correct answer The frictional force is equal to the component of the weight (Mg) parallel to the surface, which is equal to Mg sin θ.

Exercise:
Newton II - Object on a slope

Problem:

A force T = 312 N is required to keep a body at rest on a frictionless inclined plane which makes an angle of 35° with the horizontal. The forces acting on the body are shown. Calculate the magnitudes of forces P and R, giving your answers to three significant figures.



Solution:

Find the magnitude of P We are usually asked to find the magnitude of T, but in this case T is given and we are asked to find P. We can use the same equation. T is the force that balances the component of P parallel to the plane (P$_x$) and therefore it has the same magnitude.

Equation:

$$\begin{aligned} T &= P\ \sin\ \theta \\ 312 &= P\ \sin\ 35^\circ \\ P &= 544\ \text{N} \end{aligned}$$

Find the magnitude of R R can also be determined with the use of trigonometric ratios. The tan or cos ratio can be used. We recommend that you use the tan ratio because it does not involve using the value for P (for in case you made a mistake in calculating P).

Equation:

$$\begin{aligned} \tan\ 55^\circ &= \frac{R}{T} \\ \tan\ 55^\circ &= \frac{R}{312} \\ R &= \tan\ 55^\circ \times 312 \\ R &= 445,6\ \text{N} \\ R &= 446\ \text{N} \end{aligned}$$

Note that the question asks that the answers be given to 3 significant figures. We therefore round 445,6 N up to 446 N.

Lifts and rockets

So far we have looked at objects being pulled or pushed across a surface, in other words motion parallel to the surface the object rests on. Here we only considered forces parallel to the surface, but we can also lift objects up or let them fall. This is vertical motion where only vertical forces are being considered.

Let us consider a 500 kg lift, with no passengers, hanging on a cable. The purpose of the cable is to pull the lift upwards so that it can reach the next floor or to let go a little so that it can move downwards to the floor below. We will look at five possible stages during the motion of the lift.

Stage 1:

The 500 kg lift is stationary at the second floor of a tall building.

Because the lift is stationary (not moving) there is no resultant force acting on the lift. This means that the upward forces must be balanced by the downward forces. The only force acting down is the force of gravity which is equal to (500 x 9,8 = 4900 N) in this case. The cable must therefore pull upwards with a force of 4900 N to keep the lift stationary at this point.

Stage 2:

The lift moves upwards at an acceleration of 1 m·s^{-2}.

If the lift is accelerating, it means that there is a resultant force in the direction of the motion. This means that the force acting upwards is now greater than the force of gravity F$_g$ (down). To find the magnitude of the force applied by the cable (F$_c$) we can do the following calculation: (Remember to choose a direction as positive. We have chosen upwards as positive.)
Equation:

$$\begin{aligned} F_R &= ma \\ F_c + F_g &= ma \\ F_c + (-4900) &= (500)(1) \\ F_c &= 5400\text{ N upwards} \end{aligned}$$

The answer makes sense as we need a bigger force upwards to cancel the effect of gravity as well as make the lift go faster.

Stage 3:

The lift moves at a constant velocity.

When the lift moves at a constant velocity, it means that all the forces are balanced and that there is no resultant force. The acceleration is zero, therefore F$_R$ = 0. The force acting upwards is equal to the force acting downwards, therefore F$_c$ = 4900 N.

Stage 4:

The lift slows down at a rate of 2m·s^{-2}.

As the lift is now slowing down there is a resultant force downwards. This means that the force acting downwards is greater than the force acting upwards. To find the magnitude of the force applied by the cable (F$_c$) we can do the following calculation: Again we have chosen upwards as positive, which means that the acceleration will be a negative number.
Equation:

$$\begin{aligned} F_R &= ma \\ F_c + F_g &= ma \\ F_c + (-4900) &= (500)(-2) \\ F_c &= 3900\text{ N upwards} \end{aligned}$$

This makes sense as we need a smaller force upwards to ensure that the resultant force is downward. The force of gravity is now greater than the upward pull of the cable and the lift will slow down.

Stage 5:

The cable snaps.

When the cable snaps, the force that used to be acting upwards is no longer present. The only force that is present would be the force of gravity. The lift will freefall and its acceleration can be calculated as follows:

Equation:

$$\begin{aligned} F_R &= ma \\ F_c + F_g &= ma \\ 0 + (-4900) &= (500)(a) \\ a &= -9,8\,\mathrm{m} \cdot \mathrm{s}^{-2} \\ a &= 9,8\,\mathrm{m} \cdot \mathrm{s}^{-2}\ \text{downwards} \end{aligned}$$

Rockets

As with lifts, rockets are also examples of objects in vertical motion. The force of gravity pulls the rocket down while the thrust of the engine pushes the rocket upwards. The force that the engine exerts must overcome the force of gravity so that the rocket can accelerate upwards. The worked example below looks at the application of Newton's Second Law in launching a rocket.

Exercise:
Newton II - rocket

Problem:

A rocket is launched vertically upwards into the sky at an acceleration of 20 $\mathrm{m \cdot s^{-2}}$. If the mass of the rocket is 5000 kg, calculate the magnitude and direction of the thrust of the rocket's engines.

Solution:

Analyse what is given and what is asked	We have the following:	m = 5000 kg	a = 20 $\mathrm{m \cdot s^{-2}}$	F_g = 5000 x 9,8 = 49000 N	We are asked to find the thrust of the rocket engine F_1.

Find the thrust of the engine

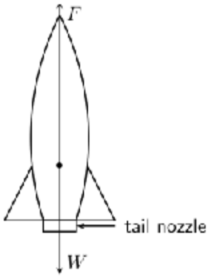
We will apply Newton's Second Law:

Equation:

$$\begin{aligned} F_R &= ma \\ F_1 + F_g &= ma \\ F_1 + (-49000) &= (5000)(20) \\ F_1 &= 149\ 000\ \mathrm{N}\ \ \text{upwards} \end{aligned}$$

Exercise:
Rockets

Problem: How do rockets accelerate in space?



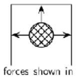
Solution:

What happens to the rocket fuel?

Gas explodes inside the rocket.

What effect does this have?

This exploding gas exerts a force on each side of the rocket (as shown in the picture below of the explosion chamber inside the rocket).



Note that the forces shown in this picture are representative. With an explosion there will be forces in all directions.

What are the forces?

Due to the symmetry of the situation, all the forces exerted on the rocket are balanced by forces on the opposite side, except for the force opposite the open side. This force on the upper surface is unbalanced.

Think about the resultant force

This is therefore the resultant force acting on the rocket and it makes the rocket accelerate forwards.

Exercise:
Newton II - lifts

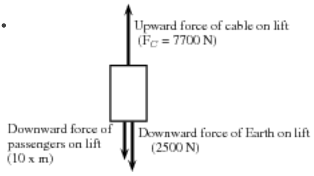
Problem:

A lift, mass 250 kg, is initially at rest on the ground floor of a tall building. Passengers with an unknown total mass, m, climb into the lift. The lift accelerates upwards at 1,6 m·s^{-2}. The cable supporting the lift exerts a constant upward force of 7700 N. Use g = 10 m·s^{-2}.

1. Draw a labeled force diagram indicating all the forces acting on the lift while it accelerates upwards.
2. What is the maximum mass, m, of the passengers the lift can carry in order to achieve a constant upward acceleration of 1,6 m·s^{-2}.

Solution:

Draw a force diagram.


Upward force of cable on lift
(F_C = 7700 N)
Downward force of passengers on lift
(10 x m)
Downward force of Earth on lift
(2500 N)

Find the mass, m.

Let us look at the lift with its passengers as a unit. The mass of this unit will be (250 + m) kg and the force of the Earth pulling downwards (F_g) will be (250 + m) x 10 m.s^{-2}. If we apply Newton's Second Law to the situation we get:

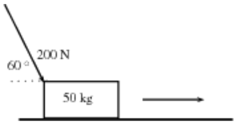
Equation:

$$\begin{aligned} F_{net} &= ma \\ F_C - F_g &= ma \\ 7700 - (250+m)(10) &= (250+m)(1,6) \\ 7700 - 2500 - 10\ m &= 400 + 1,6\ m \\ 4800 &= 11,6\ m \\ m &= 413,79 \text{ kg} \end{aligned}$$

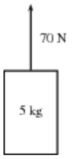
Exercise

1. A tug is capable of pulling a ship with a force of 100 kN. If two such tugs are pulling on one ship, they can produce any force ranging from a minimum of 0 N to a maximum of 200 kN. Give a detailed explanation of how this is possible. Use diagrams to support your result.

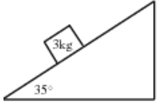
2. A car of mass 850 kg accelerates at 2 m·s^{-2}. Calculate the magnitude of the resultant force that is causing the acceleration.
3. Find the force needed to accelerate a 3 kg object at 4 m·s^{-2}.
4. Calculate the acceleration of an object of mass 1000 kg accelerated by a force of 100 N.
5. An object of mass 7 kg is accelerating at 2,5 m·s^{-2}. What resultant force acts on it?
6. Find the mass of an object if a force of 40 N gives it an acceleration of 2 m·s^{-2}.
7. Find the acceleration of a body of mass 1 000 kg that has a 150 N force acting on it.
8. Find the mass of an object which is accelerated at 2 m·s^{-2} by a force of 40 N.
9. Determine the acceleration of a mass of 24 kg when a force of 6 N acts on it. What is the acceleration if the force were doubled and the mass was halved?
10. A mass of 8 kg is accelerating at 5 m·s^{-2}.
    a. Determine the resultant force that is causing the acceleration.
    b. What acceleration would be produced if we doubled the force and reduced the mass by half?
11. A motorcycle of mass 100 kg is accelerated by a resultant force of 500 N. If the motorcycle starts from rest:
    a. What is its acceleration?
    b. How fast will it be travelling after 20 s?
    c. How long will it take to reach a speed of 35 m·s^{-1}?
    d. How far will it travel from its starting point in 15 s?
12. A force acting on a trolley on a frictionless horizontal plane causes an acceleration of magnitude 6 m·s^{-2}. Determine the mass of the trolley.
13. A force of 200 N, acting at 60° to the horizontal, accelerates a block of mass 50 kg along a horizontal plane as shown.



    1. Calculate the component of the 200 N force that accelerates the block horizontally.
    2. If the acceleration of the block is 1,5 m·s^{-2}, calculate the magnitude of the frictional force on the block.
    3. Calculate the vertical force exerted by the block on the plane.
14. A toy rocket of mass 0,5 kg is supported vertically by placing it in a bottle. The rocket is then ignited. Calculate the force that is required to accelerate the rocket vertically upwards at 8 m·s^{-2}.
15. A constant force of 70 N is applied vertically to a block of mass 5 kg as shown. Calculate the acceleration of the block.



16. A stationary block of mass 3kg is on top of a plane inclined at 35° to the horizontal.



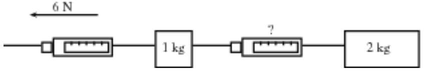
    1. Draw a force diagram (not to scale). Include the weight (F_g) of the block as well as the components of the weight that are perpendicular and parallel to the inclined plane.
    2. Determine the values of the weight's perpendicular and parallel components (F_{gx} and F_{gy}).
    3. Determine the magnitude and direction of the frictional force between the block and plane.
17. A student of mass 70 kg investigates the motion of a lift. While he stands in the lift on a bathroom scale (calibrated in newton), he notes three stages of his journey.

    1. For 2 s immediately after the lift starts, the scale reads 574 N.
    2. For a further 6 s it reads 700 N.
    3. For the final 2 s it reads 854 N.

    Answer the following questions:

    1. Is the motion of the lift upward or downward? Give a reason for your answer.
    2. Write down the magnitude and the direction of the resultant force acting on the student for each of the stages I, II and III.
    3. Calculate the magnitude of the acceleration of the lift during the first 2s.

18. A car of mass 800 kg accelerates along a level road at 4 m·s^{-2}. A frictional force of 700 N opposes its motion. What force is produced by the car's engine?
19. Two objects, with masses of 1 kg and 2 kg respectively, are placed on a smooth surface and connected with a piece of string. A horizontal force of 6 N is applied with the help of a spring balance to the 1 kg object. Ignoring friction, what will the force acting on the 2 kg mass, as measured by a second spring balance, be?

20. A rocket of mass 200 kg has a resultant force of 4000 N upwards on it.

    1. What is its acceleration in space, where it has no weight?
    2. What is its acceleration on the Earth, where it has weight?
    3. What driving force does the rocket engine need to exert on the back of the rocket in space?
    4. What driving force does the rocket engine need to exert on the back of the rocket on the Earth?

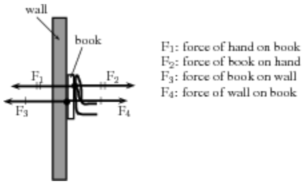
21. A car going at 20 m·s^{-1}accelerates uniformly and comes to a stop in a distance of 20 m.

    1. What is its acceleration?
    2. If the car is 1000 kg how much force do the brakes exert?

Newton's Third Law of Motion

Newton's Third Law of Motion deals with the interaction between pairs of objects. For example, if you hold a book up against a wall you are exerting a force on the book (to keep it there) and the book is exerting a force back at you (to keep you from falling through the book). This may sound strange, but if the book was not pushing back at you, your hand would push through the book! These two forces (the force of the hand on the book (F_1) and the force of the book on the hand (F_2)) are called an action-reaction pair of forces. They have the same magnitude, but act in opposite directions and act on different objects (the one force is onto the book and the other is onto your hand).

There is another action-reaction pair of forces present in this situation. The book is pushing against the wall (action force) and the wall is pushing back at the book (reaction). The force of the book on the wall (F_3) and the force of the wall on the book (F_4) are shown in the diagram.


wall
book
F1
F2
F3
F4
F1: force of hand on book
F2: force of book on hand
F3: force of book on wall
F4: force of wall on book

Newton's action-reaction pairs

Newton's Third Law of Motion

> If body A exerts a force on body B, then body B exerts a force of equal magnitude on body A, but in the opposite direction.

Khan academy video on newtons laws - 3

[missing_resource: http://www.youtube.com/v/By-ggTfeuJU&rel=0&hl=en_US&feature=player_embedded&version=3]

Newton's action-reaction pairs can be found everywhere in life where two objects interact with one another. The following worked examples will illustrate this:

Exercise:

Newton III - seat belt

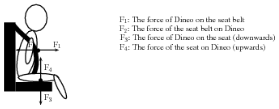
Problem:

Dineo is seated in the passenger seat of a car with the seat belt on. The car suddenly stops and he moves forwards until the seat belt stops him. Draw a labeled force diagram identifying two action-reaction pairs in this situation.

Solution:

Draw a force diagram Start by drawing the picture. You will be using arrows to indicate the forces so make your picture large enough so that detailed labels can also be added. The picture needs to be accurate, but not artistic! Use stickmen if you have to.

Label the diagram Take one pair at a time and label them carefully. If there is not enough space on the drawing, then use a key on the side.


F
F₁
F₄
F₃
F₁: The force of Dineo on the seat belt
F₂: The force of the seat belt on Dineo
F₃: The force of Dineo on the seat (downwards)
F₄: The force of the seat on Dineo (upwards)

Exercise:

Newton III - forces in a lift

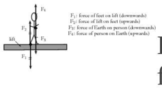
Problem:

Tammy travels from the ground floor to the fifth floor of a hotel in a lift. Which ONE of the following statements is TRUE about the force exerted by the floor of the lift on Tammy's feet?

A. It is greater than the magnitude of Tammy's weight.
B. It is equal in magnitude to the force Tammy's feet exert on the floor.
C. It is equal to what it would be in a stationary lift.
D. It is greater than what it would be in a stationary lift.

Solution:

Analyse the situation This is a Newton's Third Law question and not Newton II. We need to focus on the action-reaction pairs


F₄
F₂
F₃
lift
F₁
F₁: force of feet on lift (downwards)
F₂: force of lift on feet (upwards)
F₃: force of Earth on person (downwards)
F₄: force of person on Earth (upwards)

Newton's

In this question statements are made about the force of the floor (lift) on Tammy's feet. This force corresponds to F_2 in our diagram. The

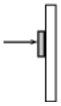
of forces and not the motion of the lift. The following diagram will show the action-reaction pairs that are present when a person is standing on a scale in a lift.	action-reaction pairs in a lift	reaction force that pairs up with this one is F_1, which is the force that Tammy's feet exerts on the floor of the lift. The magnitude of these two forces are the same, but they act in opposite directions.

Choose the correct answer	It is important to analyse the question first, before looking at the answers as the answers might confuse you. Make sure that you understand the situation and know what is asked before you look at the options.	The correct answer is B.

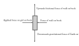
Exercise:
Newton III - book and wall

Problem: Tumi presses a book against a vertical wall as shown in the sketch.

1. Draw a labelled force diagram indicating all the forces acting on the book.
2. State, in words, Newton's Third Law of Motion.
3. Name the action-reaction pairs of forces acting in the horizontal plane.



Solution:

Draw a force diagram	A force diagram will look like this:		Note that we had to draw all the forces acting on the book and not the action-reaction pairs. None of the forces drawn are action-reaction pairs, because they all act on the same object (the book). When you label forces, be as specific as possible, including the direction of the force and both objects involved, for example, do not say gravity (which is an incomplete answer) but rather say 'Downward (direction) gravitational force of the Earth (object) on the book (object)'.	
State Newton's Third Law	If body A exerts a force onto body B, then body B will exert a force equal in magnitude, but opposite in direction, onto body A.			
Name the action-reaction pairs	The question only asks for action-reaction forces in the horizontal plane. Therefore:	Pair 1: Action: Applied force of the girl on the book; Reaction: The force of the book on the girl.	Pair 2: Action: Force of the book on the wall; Reaction: Force of the wall on the book.	Note that a Newton III pair will always involve the same combination of words, like 'book on wall' and 'wall on book'. The objects are 'swapped around' in naming the pairs.

Experiment : Balloon Rocket

Aim:

In this experiment for the entire class, you will use a balloon rocket to investigate Newton's Third Law. A fishing line will be used as a track and a plastic straw taped to the balloon will help attach the balloon to the track.

Apparatus:

You will need the following items for this experiment:

1. balloons (one for each team)
2. plastic straws (one for each team)
3. tape (cellophane or masking)
4. fishing line, 10 meters in length
5. a stopwatch - optional (a cell phone can also be used)
6. a measuring tape - optional

Method:

1. Divide into groups of at least five.
2. Attach one end of the fishing line to the blackboard with tape. Have one teammate hold the other end of the fishing line so that it is taut and roughly horizontal. The line must be held steady and **must not** be moved up or down during the experiment.
3. Have one teammate blow up a balloon and hold it shut with his or her fingers. Have another teammate tape the straw along the side of the balloon. Thread the fishing line through the straw and hold the balloon at the far end of the line.
4. Let go of the rocket and observe how the rocket moves forward.
5. Optionally, the rockets of each group can be timed to determine a winner of the fastest rocket.

    1. Assign one teammate to time the event. The balloon should be let go when the time keeper yells “Go!” Observe how your rocket moves toward the blackboard.
    2. Have another teammate stand right next to the blackboard and yell “Stop!” when the rocket hits its target. If the balloon does not make it all the way to the blackboard, “Stop!” should be called when the balloon stops moving. The timekeeper should record the flight time.
    3. Measure the exact distance the rocket traveled. Calculate the average speed at which the balloon traveled. To do this, divide the distance traveled by the time the balloon was “in flight.” Fill in your results for Trial 1 in the Table below.
    4. Each team should conduct two more races and complete the sections in the Table for Trials 2 and 3. Then calculate the average speed for the three trials to determine your team's race entry time.

Results:

	Distance (m)	**Time (s)**	**Speed ($\mathbf{m{\cdot}s^{-1}}$)**
Trial 1			
Trial 2			
Trial 3			
		Average:	

Conclusions: The winner of this race is the team with the fastest average balloon speed.

While doing the experiment, you should think about,

1. What made your rocket move?
2. How is Newton's Third Law of Motion demonstrated by this activity?
3. Draw pictures using labeled arrows to show the forces acting on the inside of the balloon before it was released and after it was released.

Exercise

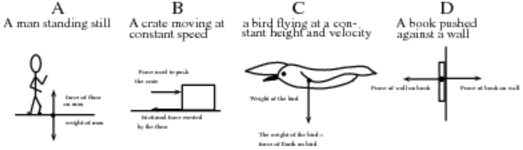
1. A fly hits the front windscreen of a moving car. Compared to the magnitude of the force the fly exerts on the windscreen, the magnitude of the force the windscreen exerts on the fly during the collision, is ...

   A. zero.
   B. smaller, but not zero.
   C. bigger.
   D. the same.

   ·gb 0.8,0.0,0.2 rgb 0.4,0.4,0.4 rgb 0.6,0.6,0.6 rgb 0.8,0.8,0.8

   ·gb 0.8,0.8,0.8 ptm m n ptm m n ptm m n ptm m n ptm m n ptm m n ptm m n ptm m n ptm m n ptm m n ptm m n pt

2. Which of the following pairs of forces correctly illustrates Newton's Third Law?


A
A man standing still
Force of floor on man
weight of man
B
A crate moving at constant speed
Force used to push the crate
frictional force exerted by the floor
C
a bird flying at a constant height and velocity
Weight of the bird
The weight of the bird = force of Earth on bird
D
A book pushed against a wall
Force of wall on book
Force of book on wall

Different types of forces

Tension

Tension is the magnitude of the force that exists in objects like ropes, chains and struts that are providing support. For example, there are tension forces in the ropes supporting a child's swing hanging from a tree.

Contact and non-contact forces

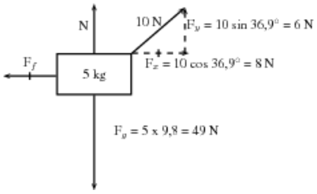
In this chapter we have come across a number of different types of forces, for example a push or a pull, tension in a string, frictional forces and the normal force. These are all examples of contact forces where there is a physical point of contact between applying the force and the object. Non-contact forces are forces that act over a distance, for example magnetic forces, electrostatic forces and gravitational forces.

When an object is placed on a surface, two types of surface forces can be identified. Friction is a force that acts between the surface and the object and is parallel to the surface. The normal force is a force that acts between the object and the surface and is perpendicular to the surface.

The normal force

A 5 kg box is placed on a rough surface and a 10 N force is applied at an angle of $36{,}9^\circ$ to the horizontal. The box does not move. The normal force (N or F$_N$) is the force between the box and the surface acting in the vertical direction. If this force is not present the box would fall through the surface because the force of gravity pulls it downwards. The normal force therefore acts upwards. We can calculate the normal force by considering all the forces in the vertical direction. All the forces in the vertical direction must add up to zero because there is no movement in the vertical direction.
Equation:

$$\begin{aligned} N + F_y + F_g &= 0 \\ N + 6 + (-49) &= 0 \\ N &= 43\,\mathrm{N}\ \text{upwards} \end{aligned}$$


N
10 N
$F_y = 10 \sin 36{,}9° = 6$ N
F_f
5 kg
$F_x = 10 \cos 36{,}9° = 8$ N
$F_g = 5 \times 9{,}8 = 49$ N

Friction and the normal force

The most interesting and illustrative normal force question, that is often asked, has to do with a scale in a lift. Using Newton's third law we can solve these problems quite easily.

When you stand on a scale to measure your weight you are pulled down by gravity. There is no acceleration downwards because there is a reaction force we call the normal force acting upwards on you. This is the force that the scale would measure. If the gravitational force were less then the reading on the scale would be less.

Exercise:

Normal Forces 1

Problem:

A man with a mass of 100 kg stands on a scale (measuring newtons). What is the reading on the scale?

Solution:

Identify what information is given and what is asked for

We are given the mass of the man. We know the gravitational acceleration that acts on him is 9,8 = m·s^{-2}.

Decide what equation to use to solve the problem

The scale measures the normal force on the man. This is the force that balances gravity. We can use Newton's laws to solve the problem:

Equation:

$$F_r = F_g + F_N$$

where F_r is the resultant force on the man.

Firstly we determine the force on the man due to gravity

Equation:

$$\begin{aligned} F_g &= mg \\ &= (100\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right) \\ &= 980\ \ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2} \\ &= 980\,\text{N}\ \ \text{downwards} \end{aligned}$$

Now determine the normal force acting upwards on the man

We now know the gravitational force downwards. We know that the sum of all the forces must equal the resultant acceleration times the mass. The overall resultant acceleration of the man on the scale is 0 - so $F_r = 0$.

Equation:

$$\begin{aligned} F_r &= F_g + F_N \\ 0 &= -980\,\text{N} + \text{F}_\text{N} \\ F_N &= 980\,\text{N}\ \ \text{upwards} \end{aligned}$$

Quote the final answer

The normal force is 980 N upwards. It exactly balances the gravitational force downwards so there is no net force and no acceleration on the man. The reading on the scale is 980 N.

Now we are going to add things to exactly the same problem to show how things change slightly. We will now move to a lift moving at constant velocity. Remember if velocity is constant then acceleration is zero.

Exercise:
Normal Forces 2

Problem:

A man with a mass of 100 kg stands on a scale (measuring newtons) inside a lift that is moving downwards at a constant velocity of 2 m·s^{-1}. What is the reading on the scale?

Solution:

Identify what information is given and what is asked for

We are given the mass of the man and the acceleration of the lift. We know the gravitational acceleration that acts on him.

Decide which equation to use to solve the problem

Once again we can use Newton's laws. We know that the sum of all the forces must equal the resultant force, F_r.

Equation:

$$F_r = F_g + F_N$$

Determine the force due to gravity

Equation:

$$\begin{aligned} F_g &= mg \\ &= (100\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right) \\ &= 980\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2} \\ &= 980\,\text{N}\ \ \text{downwards} \end{aligned}$$

Now determine the normal force acting upwards on the man

The scale measures this normal force, so once we have determined it we will know the reading on the scale. Because the lift is moving at constant velocity the overall resultant acceleration of the man on the scale is 0. If we write out the equation:

Equation:

$$\begin{aligned} F_r &= F_g + F_N \\ ma &= F_g + F_N \\ (100)(0) &= -980\,\text{N} + \text{F}_\text{N} \end{aligned}$$

$$F_N = 980\,\text{N}\ \ \text{upwards}$$

Quote the final answer

The normal force is 980 N upwards. It exactly balances the gravitational force downwards so there is no net force and no acceleration on the man. The reading on the scale is 980 N.

In the previous two examples we got exactly the same result because the net acceleration on the man was zero! If the lift is accelerating downwards things are slightly different and now we will get a more interesting answer!

Exercise:
Normal Forces 3

Problem:

A man with a mass of 100 kg stands on a scale (measuring newtons) inside a lift that is accelerating downwards at 2 m·s^{-2}. What is the reading on the scale?

Solution:

Identify what information is given and what is asked for

We are given the mass of the man and his resultant acceleration - this is just the acceleration of the lift. We know the gravitational acceleration also acts on him.

Decide which equation to use to solve the problem

Once again we can use Newton's laws. We know that the sum of all the forces must equal the resultant force, F_r.

Equation:

$$F_r = F_g + F_N$$

Determine the force due

Equation:

to gravity,

$$\begin{aligned}F_g &= mg\\ &= (100\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right)\\ &= 980\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2}\\ &= 980\,\text{N}\ \ \text{downwards}\end{aligned}$$

Determine the resultant force, The resultant force can be calculated by applying Newton's Second Law:

Equation:

$$\begin{aligned}F_r &= ma\\ F_r &= (100\ \text{kg})\left(-2\,\text{m}\cdot\text{s}^{-2}\right)\\ &= -200\ N\\ &= 200\ N\ \ \text{down}\end{aligned}$$

Determine the normal force, The sum of all the vertical forces is equal to the resultant force, therefore

Equation:

$$\begin{aligned}F_r &= F_g + F_N\\ -200\,\text{N} &= -980\,\text{N} + \text{F}_\text{N}\\ F_N &= 780\,\text{N}\ \ \text{upwards}\end{aligned}$$

Quote the final answer The normal force is 780 N upwards. It balances the gravitational force downwards just enough so that the man only accelerates downwards at 2 m·s^{-2}. The reading on the scale is 780 N.

Exercise:
Normal Forces 4

Problem:

A man with a mass of 100 kg stands on a scale (measuring newtons) inside a lift that is accelerating upwards at 4 m·s^{-2}. What is the reading on the scale?

Solution:

Identify what information is given and what is asked for We are given the mass of the man and his resultant acceleration - this is just the acceleration of the lift. We know the gravitational acceleration also acts on him.

Decide which equation to use to solve the problem Once again we can use Newton's laws. We know that the sum of all the forces must equal the resultant force, F_r.

Equation:

$$F_r = F_g + F_N$$

Determine the force due to gravity, **Equation:**

$$\begin{aligned}F_g &= mg\\ &= (100\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right)\\ &= 980\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2}\\ &= 980\,\text{N}\ \ \text{downwards}\end{aligned}$$

Determine the resultant force, The resultant force can be calculated by applying Newton's Second Law:

Equation:

$$\begin{aligned}F_r &= ma\\ F_r &= (100\ \text{kg})\left(4\,\text{m}\cdot\text{s}^{-2}\right)\\ &= 400\ N\ \ \text{upwards}\end{aligned}$$

Determine the **Equation:**

normal force, The sum of all the vertical forces is equal to the resultant force, therefore

$$\begin{aligned} F_r &= F_g + F_N \\ 400\,\mathrm{N} &= -980\,\mathrm{N} + \mathrm{F_N} \\ F_N &= 1380\,\mathrm{N}\ \ \text{upwards} \end{aligned}$$

Quote the final answer The normal force is 1380 N upwards. It balances the gravitational force downwards and then in addition applies sufficient force to accelerate the man upwards at 4m·s^{-2}. The reading on the scale is 1380 N.

Friction forces

When the surface of one object slides over the surface of another, each body exerts a frictional force on the other. For example if a book slides across a table, the table exerts a frictional force onto the book and the book exerts a frictional force onto the table (Newton's Third Law). Frictional forces act parallel to surfaces.

A force is not always powerful enough to make an object move, for example a small applied force might not be able to move a heavy crate. The frictional force opposing the motion of the crate is equal to the applied force but acting in the opposite direction. This frictional force is called *static friction*. When we increase the applied force (push harder), the frictional force will also increase until the applied force overcomes it. This frictional force can vary from zero (when no other forces are present and the object is stationary) to a maximum that depends on the surfaces. When the applied force is greater than the maximum frictional force, the crate will move. Once the object moves, the frictional force will decrease and remain at that level, which is also dependent on the surfaces, while the objects are moving. This is called *kinetic friction*. In both cases the maximum frictional force is related to the normal force and can be calculated as follows:

For static friction: F$_f \leq \mu_s$ N

Where μ_s = the coefficient of static friction

and N = normal force

For kinetic friction: F$_f$ = μ_k N

Where μ_k = the coefficient of kinetic friction

and N = normal force

Remember that static friction is present when the object is not moving and kinetic friction while the object is moving. For example when you drive at constant velocity in a car on a tar road you have to keep the accelerator pushed in slightly to overcome the friction between the tar road and the wheels of the car. However, while moving at a constant velocity the wheels of the car are rolling, so this is not a case of two surfaces “rubbing” against eachother and we are in fact looking at static friction. If you should break hard, causing the car to skid to a halt, we would be dealing with two surfaces rubbing against eachother and hence kinetic friction. The higher the value for the coefficient of friction, the more 'sticky' the surface is and the lower the value, the more 'slippery' the surface is.

The frictional force (F$_f$) acts in the horizontal direction and can be calculated in a similar way to the normal for as long as there is no movement. If we use the same example as in [link] and we choose to the rightward direction as positive,

Equation:

$$
\begin{aligned}
F_f + F_x &= 0 \\
F_f + (+8) &= 0 \\
F_f &= -8 \\
F_f &= 8\,\mathrm{N}\ \text{to the left}
\end{aligned}
$$

Exercise:
Forces on a slope

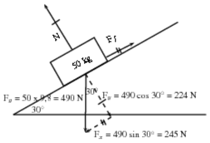
Problem:

A 50 kg crate is placed on a slope that makes an angle of 30° with the horizontal. The box does not slide down the slope. Calculate the magnitude and direction of the frictional force and the normal force present in this situation.

Solution:

Draw a force diagram

Draw a force diagram and fill in all the details on the diagram. This makes it easier to understand the problem.


N
Ff
50 kg
$F_g = 50 \times 9,8 = 490$ N
30°
30°
$F_y = 490 \cos 30^\circ = 224$ N
$F_x = 490 \sin 30^\circ = 245$ N

Friction and the normal forces on a slope

Calculate the normal force

The normal force acts perpendicular to the surface (and not vertically upwards). It's magnitude is equal to the component of the weight perpendicular to the slope. Therefore:

Equation:

$$
\begin{aligned}
N &= F_g\ cos\ 30^\circ \\
N &= 490\ cos\ 30^\circ \\
N &= 224\,\mathrm{N}\ \text{perpendicular to the surface}
\end{aligned}
$$

Calculate the frictional force

The frictional force acts parallel to the sloped surface. It's magnitude is equal to the component of the weight parallel to the slope. Therefore:

Equation:

$$
\begin{aligned}
F_f &= F_g\ sin\ 30^\circ \\
F_f &= 490\ sin\ 30^\circ \\
F_f &= 245\,\mathrm{N}\ \text{up the slope}
\end{aligned}
$$

We often think about friction in a negative way but very often friction is useful without us realizing it. If there was no friction and you tried to prop a ladder up against a wall, it would simply slide to the ground. Rock climbers use friction to maintain their grip on cliffs. The brakes of cars would be useless if it wasn't for friction!

Exercise:
Coefficients of friction

Problem:

A block of wood weighing 32 N is placed on a rough, flat inclline and a rope is tied to it. The tension in the rope can be increased to 8 N before the block starts to slide. A force of 4 N will keep the block moving at constant speed once it has been set in motion. Determine the coefficients of static and kinetic friction.

Solution:

Analyse the question and determine what is asked

The weight of the block is given (32 N) and two situations are identified: One where the block is not moving (applied force is 8 N), and one where the block is moving (applied force is 4 N).

We are asked to find the coefficient for static friction μ_s and kinetic friction μ_k.

Find the coefficient of static friction

Equation:

$$
\begin{aligned}
F_f &= \mu_s N \\
8 &= \mu_s (32) \\
\mu_s &= 0,25
\end{aligned}
$$

Note that the coefficient of friction does not have a unit as it shows a ratio. The value for the coefficient of friction friction can have any value up to a maximum of 0,25. When a force less than 8 N is applied, the coefficient of friction will be less than 0,25.

Find the coefficient of kinetic friction

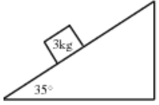
The coefficient of kinetic friction is sometimes also called the coefficient of dynamic friction. Here we look at when the block is moving:

Equation:

$$
\begin{aligned}
F_f &= \mu_k N \\
4 &= \mu_k (32) \\
\mu_k &= 0,125
\end{aligned}
$$

Exercise

1. A 12 kg box is placed on a rough surface. A force of 20 N applied at an angle of 30° to the horizontal cannot move the box. Calculate the magnitude and direction of the normal and friction forces.
2. A 100 kg crate is placed on a slope that makes an angle of 45° with the horizontal. The box does not slide down the slope. Calculate the magnitude and direction of the frictional force and the normal force present in this situation.
3. What force T at an angle of 30° above the horizontal, is required to drag a block weighing 20 N to the right at constant speed, if the coefficient of kinetic friction between the block and the surface is 0,20?
4. A block weighing 20 N rests on a horizontal surface. The coefficient of static friction between the block and the surface is 0,40 and the coefficient of dynamic friction is 0,20.
    1. What is the magnitude of the frictional force exerted on the block while the block is at rest?
    2. What will the magnitude of the frictional force be if a horizontal force of 5 N is exerted on the block?
    3. What is the minimum force required to start the block moving?
    4. What is the minimum force required to keep the block in motion once it has been started?
    5. If the horizontal force is 10 N, determine the frictional force.
5. A stationary block of mass 3kg is on top of a plane inclined at 35° to the horizontal.



    1. Draw a force diagram (not to scale). Include the weight of the block as well as the components of the weight that are perpendicular and parallel to the inclined plane.
    2. Determine the values of the weight's perpendicular and parallel components.
    3. There exists a frictional force between the block and plane. Determine this force (magnitude and direction).
6. A lady injured her back when she slipped and fell in a supermarket. She holds the owner of the supermarket accountable for her medical expenses. The owner claims that the floor covering was not wet and meets the accepted standards. He therefore cannot accept responsibility. The matter eventually ends up in court. Before passing judgement, the judge approaches you, a science student, to determine whether the coefficient of static friction of the floor is a minimum of 0,5 as required. He provides you with a tile from the floor, as well as one of the shoes the lady was wearing on the day of the incident.

1. Write down an expression for the coefficient of static friction.
2. Plan an investigation that you will perform to assist the judge in his judgement. Follow the steps outlined below to ensure that your plan meets the requirements.
    1. Formulate an investigation question.
    2. Apparatus: List *all* the other apparatus, except the tile and the shoe, that you will need.
    3. A stepwise method: How will you perform the investigation? Include a relevant, labelled free body-diagram.
    4. Results: What will you record?
    5. Conclusion: How will you interpret the results to draw a conclusion?

Forces in equilibrium

At the beginning of this chapter it was mentioned that resultant forces cause objects to accelerate in a straight line. If an object is stationary or moving at constant velocity then either,

- no forces are acting on the object, or
- the forces acting on that object are exactly balanced.

In other words, for stationary objects or objects moving with constant velocity, the resultant force acting on the object is zero. Additionally, if there is a perpendicular moment of force, then the object will rotate. You will learn more about moments of force later in this chapter.

Therefore, in order for an object not to move or to be in *equilibrium*, the sum of the forces (resultant force) must be zero and the sum of the moments of force must be zero.

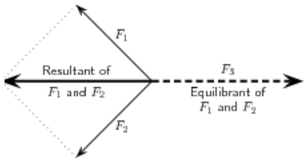
Equilibrium

> An object in equilibrium has both the sum of the forces acting on it and the sum of the moments of the forces equal to zero.

If a resultant force acts on an object then that object can be brought into equilibrium by applying an additional force that exactly balances this resultant. Such a force is called the *equilibrant* and is equal in magnitude but opposite in direction to the original resultant force acting on the object.

Equilibrant

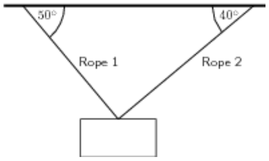
> The equilibrant of any number of forces is the single force required to produce equilibrium, and is equal in magnitude but opposite in direction to the resultant force.



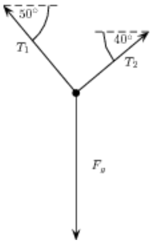
In the figure the resultant of F_1 and F_2 is shown. The equilibrant of F_1 and F_2 is then the vector opposite in direction to this resultant with the same magnitude (i.e. F_3).

- F_1, F_2 and F_3 are in equilibrium
- F_3 is the equilibrant of F_1 and F_2
- F_1 and F_2 are kept in equilibrium by F_3

As an example of an object in equilibrium, consider an object held stationary by two ropes in the arrangement below:

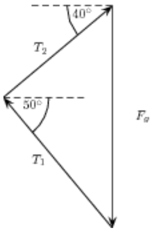


Let us draw a free body diagram for the object. In the free body diagram the object is drawn as a dot and all forces acting on the object are drawn in the correct directions starting from that dot. In this case, three forces are acting on the object.



Each rope exerts a force on the object in the direction of the rope away from the object. These tension forces are represented by T_1 and T_2. Since the object has mass, it is attracted towards the centre of the Earth. This weight is represented in the force diagram as F_g.

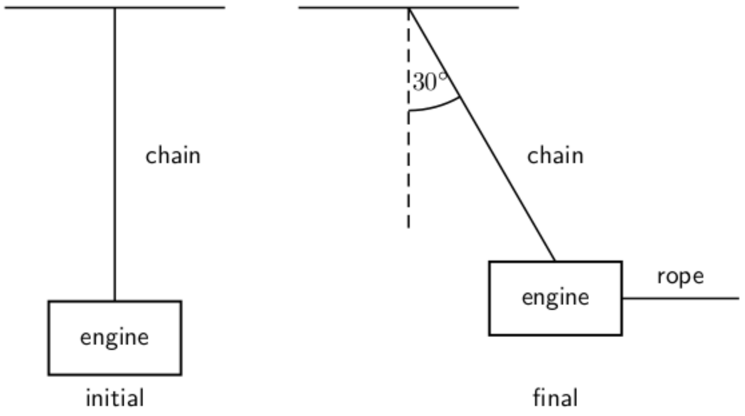
Since the object is stationary, the resultant force acting on the object is zero. In other words the three force vectors drawn tail-to-head form a closed triangle:



Exercise: Equilibrium

Problem:

A car engine of weight 2000 N is lifted by means of a chain and pulley system. The engine is initially suspended by the chain, hanging stationary. Then, the engine is pulled sideways by a mechanic, using a rope. The engine is held in such a position that the chain makes an angle of 30° with the vertical. In the questions that follow, the masses of the chain and the rope can be ignored.

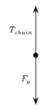


1. Draw a free body representing the forces acting on the engine in the initial situation.
2. Determine the tension in the chain initially.
3. Draw a free body diagram representing the forces acting on the engine in the final situation.
4. Determine the magnitude of the applied force and the tension in the chain in the final situations.

Solution:

Initial free body diagram for the engine

There are only two forces acting on the engine initially: the tension in the chain, T_{chain} and the weight of the engine, F_g.



Determine the tension in the chain

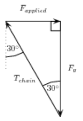
The engine is initially stationary, which means that the resultant force on the engine is zero. There are also no moments of force. Thus the tension in the chain exactly balances the weight of the engine. The tension in the chain is:

Equation:

$$\begin{aligned} T_{chain} &= F_g \\ &= 2000\,\text{N} \end{aligned}$$

Final free body diagram for the engine

There are three forces acting on the engine finally: The tension in the chain, the applied force and the weight of the engine.



Calculate the magnitude of the applied force and the tension in the chain in the final situation

Since no method was specified let us calculate the magnitudes algebraically. Since the triangle formed by the three forces is a right-angle triangle this is easily done:

Equation:

$$\begin{aligned} \frac{F_{applied}}{F_g} &= \tan 30^\circ \\ F_{applied} &= (2000\,\text{N}) \tan 30^\circ \\ &= 1\,155\,\text{N} \end{aligned}$$

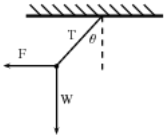
and

Equation:

$$\begin{aligned} \frac{T_{chain}}{F_g} &= \frac{1}{\cos 30^\circ} \\ T_{chain} &= \frac{2000\,\text{N}}{\cos 30^\circ} \\ &= 2\,309\,\text{N} \end{aligned}$$

Exercise

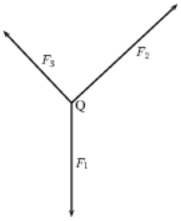
1. The diagram shows an object of weight W, attached to a string. A horizontal force F is applied to the object so that the string makes an angle of θ with the vertical when the object is at rest. The force exerted by the string is T. Which one of the following expressions is incorrect?



   1. **A**F + T + W = 0
   2. **B**W = T cos θ
   3. **C**tan θ = $\frac{F}{W}$
   4. **D**W = T sin θ

2. The point Q is in equilibrium due to three forces F_1, F_2 and F_3 acting on it. Which of the statements about these forces is INCORRECT?

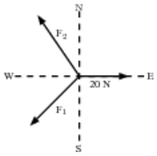
   1. **A**The sum of the forces F_1, F_2 and F_3 is zero.
   2. **B**The three forces all lie in the same plane.
   3. **C**The resultant force of F_1 and F_3 is F_2.
   4. **D**The sum of the components of the forces in any direction is zero.



3. A point is acted on by two forces in equilibrium. The forces

   1. **A**have equal magnitudes and directions.
   2. **B**have equal magnitudes but opposite directions.
   3. **C**act perpendicular to each other.
   4. **D**act in the same direction.

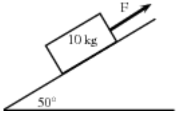
4. A point in equilibrium is acted on by three forces. Force F_1 has components 15 N due south and 13 N due west. What are the components of force F_2?



   1. **A**13 N due north and 20 due west
   2. **B**13 N due north and 13 N due west
   3. **C**15 N due north and 7 N due west
   4. **D**15 N due north and 13 N due east

5.
   1. Define the term 'equilibrant'.
   2. Two tugs, one with a pull of 2500 N and the other with a pull of 3 000 N are used to tow an oil drilling platform. The angle between the two cables is 30 °. Determine, either by scale diagram or by calculation (a clearly labelled rough sketch must be given), the equilibrant of the two forces.

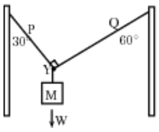
6. A 10 kg block is held motionless by a force F on a frictionless plane, which is inclined at an angle of 50° to the horizontal, as shown below:



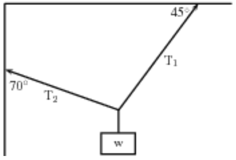
   1. Draw a force diagram (not a triangle) indicating all the forces acting on the block.
   2. Calculate the magnitude of force F. Include a labelled diagram showing a triangle of forces in your answer.

7. A rope of negligible mass is strung between two vertical struts. A mass M of weight W hangs from the rope through a hook fixed at point Y

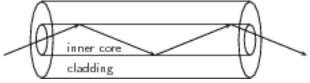
   1. Draw a vector diagram, plotted head to tail, of the forces acting at point Y. Label each force and show the size of each angle.
   2. Where will the tension be greatest? Part P or Q? Motivate your answer.
   3. When the tension in the rope is greater than 600N it will break. What is the maximum mass that the above set up can support?



8. An object of weight w is supported by two cables attached to the ceiling and wall as shown. The tensions in the two cables are T_1 and T_2 respectively. Tension $T_1 = 1200$ N. Determine the tension T_2 and weight w of the object by accurate construction and measurement or calculation.



9. A rope is tied at two points which are 70 cm apart from each other, on the same horizontal line. The total length of rope is 1 m, and the maximum tension it can withstand is 1000 N. Find the largest mass (m), in kg, that can be carried at the midpoint of the rope, without breaking the rope. Include a labelled diagram showing the triangle of forces in your answer.



Forces between Masses

In Grade 10, you saw that gravitational fields exert forces on masses in the field. A field is a region of space in which an object experiences a force. The strength of a field is defined by a field strength. For example, the gravitational field strength, g, on or near the surface of the Earth has a value that is approximately 9,8 $\text{m}\cdot\text{s}^{-2}$.

The force exerted by a field of strength g on an object of mass m is given by:
Equation:

$$F = m \cdot g$$

This can be re-written in terms of g as:
Equation:

$$g = \frac{F}{m}$$

This means that g can be understood to be a measure of force exerted per unit mass.

The force defined in [link] is known as weight.

Objects in a gravitational field exert forces on each other without touching. The gravitational force is an example of a non-contact force.

Gravity is a force and therefore must be described by a vector - so remember thta gravity has both magnitude and direction.

Newton's Law of Universal Gravitation

Newton's Law of Universal Gravitation

> Every point mass attracts every other point mass by a force directed along the line connecting the two. This force is proportional to the product of the masses and inversely proportional to the square of the distance between them.

The magnitude of the attractive gravitational force between the two point masses, F is given by:
Equation:

$$F = G\frac{m_1 m_2}{r^2}$$

where: G is the gravitational constant, m_1 is the mass of the first point mass, m_2 is the mass of the second point mass and r is the distance between the two point masses.

Assuming SI units, F is measured in newtons (N), m_1 and m_2 in kilograms (kg), r in meters (m), and the constant G is approximately equal to $6,67\times 10^{-11} N\cdot m^2\cdot\ kg^{-2}$. Remember that this is a force of attraction.

For example, consider a man of mass 80 kg standing 10 m from a woman with a mass of 65 kg. The attractive gravitational force between them would be:
Equation:

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= \left(6,67\times 10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2\cdot\ \mathrm{kg}^{-2}\right)\left(\frac{(80\,\mathrm{kg})(65\,\mathrm{kg})}{(10\mathrm{m})^2}\right) \\ &= 3,47\times 10^{-9}\,\mathrm{N} \end{aligned}$$

If the man and woman move to 1 m apart, then the force is:
Equation:

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= \left(6,67\times 10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2\cdot\ \mathrm{kg}^{-2}\right)\left(\frac{(80\,\mathrm{kg})(65\,\mathrm{kg})}{(1\mathrm{m})^2}\right) \\ &= 3,47\times 10^{-7}\,\mathrm{N} \end{aligned}$$

As you can see, these forces are very small.

Now consider the gravitational force between the Earth and the Moon. The mass of the Earth is $5,98\times 10^{24}$ kg, the mass of the Moon is $7,35\times 10^{22}$ kg and the Earth and Moon are $3,8\times 10^{8}$ m apart. The gravitational force between the Earth and Moon is:
Equation:

$$\begin{aligned} F &= G\frac{m_1 m_2}{r^2} \\ &= \left(6,67\times 10^{-11}\,\mathrm{N}\cdot\mathrm{m}^2\cdot\ \mathrm{kg}^{-2}\right)\left(\frac{\left(5,98\times 10^{24}\,\mathrm{kg}\right)\left(7,35\times 10^{22}\,\mathrm{kg}\right)}{\left(0,38\times 10^{9}\mathrm{m}\right)^2}\right) \\ &= 2,03\times 10^{20}\,\mathrm{N} \end{aligned}$$

From this example you can see that the force is very large.

These two examples demonstrate that the greater the masses, the greater the force between them. The $1/r^2$ factor tells us that the distance between the two bodies plays a role as well. The closer two bodies are, the stronger the gravitational force between them is. We feel the gravitational attraction of the Earth most at the surface since that is the closest we can get to it, but if we were in outer-space, we would barely feel the effect of the Earth's gravity!

Remember that
Equation:

$$F = m\cdot a$$

which means that every object on Earth feels the same gravitational acceleration! That means whether you drop a pen or a book (from the same height), they will both take the same length of time to hit the ground... in fact they will be head to head for the entire fall if you drop them at the same time. We can show this easily by using the two equations above (Equations [link] and [link]). The force between the Earth (which has the mass m_e) and an object of mass m_o is
Equation:

$$F = \frac{Gm_o m_e}{r^2}$$

and the acceleration of an object of mass m_o (in terms of the force acting on it) is
Equation:

$$a_o = \frac{F}{m_o}$$

So we substitute equation [link] into Equation [link], and we find that
Equation:

$$a_o = \frac{Gm_e}{r^2}$$

Since it doesn't matter what m_o is, this tells us that the acceleration on a body (due to the Earth's gravity) does not depend on the mass of the body. Thus all objects experience the same gravitational acceleration. The force on different bodies will be different but the acceleration will be the same. Due to the fact that this acceleration caused by gravity is the same on all objects we label it differently, instead of using a we use g which we call the gravitational acceleration.

Comparative Problems

Comparative problems involve calculation of something in terms of something else that we know. For example, if you weigh 490 N on Earth and the gravitational acceleration on Venus is 0,903 that of the gravitational acceleration on the Earth, then you would weigh 0,903 x 490 N = 442,5 N on Venus.

Principles for answering comparative problems

- Write out equations and calculate all quantities for the given situation
- Write out all relationships between variable from first and second case
- Write out second case
- Substitute all first case variables into second case
- Write second case in terms of first case

Exercise:
Comparative Problem 1

Problem:

A man has a mass of 70 kg. The planet Zirgon is the same size as the Earth but has twice the mass of the Earth. What would the man weigh on Zirgon, if the gravitational acceleration on Earth is 9,8 m·s^{-2}?

Solution:

Determine what information has been given

The following has been provided:

- the mass of the man, m
- the mass of the planet Zirgon (m_Z) in terms of the mass of the Earth (m_E), $m_Z = 2m_E$
- the radius of the planet Zirgon (r_Z) in terms of the radius of the Earth (r_E), $r_Z = r_E$

Determine how to approach the problem

We are required to determine the man's weight on Zirgon (w_Z). We can do this by using:

Equation:

$$w = mg = G\frac{m_1 \cdot m_2}{r^2}$$

to calculate the weight of the man on Earth and then use this value to determine the weight of the man on Zirgon.

Situation on Earth

Equation:

$$\begin{aligned} w_E &= mg_E = G\frac{m_E \cdot m}{r_E^2} \\ &= (70\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right) \\ &= 686\,\text{N} \end{aligned}$$

Situation on Zirgon in terms of situation on Earth

Write the equation for the gravitational force on Zirgon and then substitute the values for m_Z and r_Z, in terms of the values for the Earth.

Equation:

$$\begin{aligned} w_Z = mg_Z &= G\frac{m_Z \cdot m}{r_Z^2} \\ &= G\frac{2m_E \cdot m}{r_E^2} \\ &= 2\left(G\frac{m_E \cdot m}{r_E^2}\right) \\ &= 2w_E \\ &= 2(686\,\text{N}) \\ &= 1\,372\,\text{N} \end{aligned}$$

Quote the final answer

The man weighs 1 372 N on Zirgon.

Exercise:
Comparative Problem 2

Problem:

A man has a mass of 70 kg. On the planet Beeble how much will he weigh if Beeble has mass half of that of the Earth and a radius one quarter that of the Earth. Gravitational acceleration on Earth is 9,8 m·s^{-2}.

Solution:

Determine what information has been given

The following has been provided:

- the mass of the man on Earth, m
- the mass of the planet Beeble (m_B) in terms of the mass of the Earth (m_E), $m_B = \frac{1}{2}m_E$
- the radius of the planet Beeble (r_B) in terms of the radius of the Earth (r_E), $r_B = \frac{1}{4}r_E$

Determine how to approach the problem

We are required to determine the man's weight on Beeble (w_B). We can do this by using:

Equation:

$$w = mg = G\frac{m_1 \cdot m_2}{r^2}$$

to calculate the weight of the man on Earth and then use this value to determine the weight of the man on Beeble.

Situation on Earth

Equation:

$$\begin{aligned} w_E &= mg_E = G\frac{m_E \cdot m}{r_E^2} \\ &= (70\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right) \\ &= 686\,\text{N} \end{aligned}$$

Situation on Beeble in terms of situation on Earth

Write the equation for the gravitational force on Beeble and then substitute the values for m_B and r_B, in terms of the values for the Earth.

Equation:

$$\begin{aligned} w_B = mg_B &= G\frac{m_B \cdot m}{r_B^2} \\ &= G\frac{\frac{1}{2}m_E \cdot m}{\left(\frac{1}{4}r_E\right)^2} \\ &= 8\left(G\frac{m_E \cdot m}{r_E^2}\right) \\ &= 8w_E \\ &= 8(686\,\text{N}) \\ &= 5\,488\,\text{N} \end{aligned}$$

Quote the final answer

The man weighs 5 488 N on Beeble.

Physics is all about being simple - all we do is look at the world around us and notice how it really works. It is the one thing everyone is qualified to do - we spend most of our time when we are really young experimenting to find out how things work.

The actual force of air resistance is quite complicated. Experiment by moving a book through the air with the face of the book and then the side of the book forward, you will agree that the area of the book makes a difference as to how much you must work in order to move the book at the same speed in both cases. This is why racing cars are slim-lined in design, and not shaped like a big box!

Exercise

1. Two objects of mass 2m and 3m respectively exert a force F on each other when they are a certain distance apart. What will be the force between two objects situated the same distance apart but having a mass of 5m and 6m respectively?

    1. **A**0,2 F
    2. **B**1,2 F
    3. **C**2,2 F
    4. **D**5 F

2. As the distance of an object above the surface of the Earth is greatly increased, the weight of the object would

    1. **A**increase
    2. **B**decrease
    3. **C**increase and then suddenly decrease
    4. **D**remain the same

3. A satellite circles around the Earth at a height where the gravitational force is a factor 4 less than at the surface of the Earth. If the Earth's radius is R, then the height of the satellite above the surface is:

    1. **A**R
    2. **B**2 R
    3. **C**4 R
    4. **D**16 R

4. A satellite experiences a force F when at the surface of the Earth. What will be the force on the satellite if it orbits at a height equal to the diameter of the Earth:

    1. **A** $\frac{1}{F}$
    2. **B** $\frac{1}{2}F$
    3. **C** $\frac{1}{3}F$
    4. **D** $\frac{1}{9}F$

5. The weight of a rock lying on surface of the Moon is W. The radius of the Moon is R. On planet Alpha, the same rock has weight 8W. If the radius of planet Alpha is half that of the Moon, and the mass of the Moon is M, then the mass, in kg, of planet Alpha is:

    1. **A** $\frac{M}{2}$
    2. **B** $\frac{M}{4}$
    3. **C**2 M
    4. **D**4 M

6. Consider the symbols of the two physical quantities g and G used in Physics.

    1. Name the physical quantities represented by g and G.
    2. Derive a formula for calculating g near the Earth's surface using Newton's Law of Universal Gravitation. M and R represent the mass and radius of the Earth respectively.

7. Two spheres of mass 800g and 500g respectively are situated so that their centers are 200 cm apart. Calculate the gravitational force between them.
8. Two spheres of mass 2 kg and 3 kg respectively are situated so that the gravitational force between them is 2,5 x 10^{-8} N. Calculate the distance between them.
9. Two identical spheres are placed 10 cm apart. A force of 1,6675 x 10^{-9} N exists between them. Find the masses of the spheres.

10. Halley's comet, of approximate mass 1 x 10^{15} kg was 1,3 x 10^{8} km from the Earth, at its point of closest approach during its last sighting in 1986.
    1. Name the force through which the Earth and the comet interact.
    2. Is the magnitude of the force experienced by the comet the same, greater than or less than the force experienced by the Earth? Explain.
    3. Does the acceleration of the comet increase, decrease or remain the same as it moves closer to the Earth? Explain.
    4. If the mass of the Earth is 6 x 10^{24} kg, calculate the magnitude of the force exerted by the Earth on Halley's comet at its point of closest approach.

Momentum and Impulse

Momentum is a physical quantity which is closely related to forces. Momentum is a property which applies to moving objects.

Momentum

> Momentum is the tendency of an object to continue to move in its direction of travel. Momentum is calculated from the product of the mass and velocity of an object.

The momentum (symbol p) of an object of mass m moving at velocity v is:
Equation:

$$p = m \cdot v$$

According to this equation, momentum is related to both the mass and velocity of an object. A small car travelling at the same velocity as a big truck will have a smaller momentum than the truck. The smaller the mass, the smaller the velocity.

A car travelling at 120 km·hr^{-1}will have a bigger momentum than the same car travelling at 60 km·hr^{-1}. Momentum is also related to velocity; the smaller the velocity, the smaller the momentum.

Different objects can also have the same momentum, for example a car travelling slowly can have the same momentum as a motor cycle travelling relatively fast. We can easily demonstrate this. Consider a car of mass 1 000 kg with a velocity of 8 m·s^{-1}(about 30 km·hr^{-1}). The momentum of the car is therefore
Equation:

$$\begin{array}{rcl} p & = & m \cdot v \\ & = & (1000\ \text{kg})\left(8\,\text{m}\cdot\text{s}^{-1}\right) \\ & = & 8000\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1} \end{array}$$

Now consider a motor cycle of mass 250 kg travelling at 32 m·s^{-1} (about 115 km·hr^{-1}). The momentum of the motor cycle is:
Equation:

$$\begin{array}{rcl} p & = & m \cdot v \\ & = & (250\ \text{kg})\left(32\,\text{m}\cdot\text{s}^{-1}\right) \\ & = & 8000\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1} \end{array}$$

Even though the motor cycle is considerably lighter than the car, the fact that the motor cycle is travelling much faster than the car means that the momentum of both vehicles is the same.

From the calculations above, you are able to derive the unit for momentum as kg·m·s^{-1}.

Momentum is also vector quantity, because it is the product of a scalar (m) with a vector (v).

This means that whenever we calculate the momentum of an object, we need to include the direction of the momentum.

Khan academy video on momentum - 1

[missing_resource: http://www.youtube.com/v/XFhntPxow0U&rel=0&hl=en_US&feature=player_embedded&version=3]

Exercise:
Momentum of a Soccer Ball

Problem:

A soccer ball of mass 420 g is kicked at 20 m·s^{-1} towards the goal post. Calculate the momentum of the ball.

Solution:

Identify what information is given and what is asked for	The question explicitly gives	○ the mass of the ball, and ○ the velocity of the ball	The mass of the ball must be converted to SI units.	**Equation:** $420\ \text{g} = 0{,}42\ \text{kg}$	We are asked to calculate the momentum of the ball. From the definition of momentum,	**Equation:** $p = m \cdot v$	we see that we need the mass and velocity of the ball, which we are given.
Do the calculation	We calculate the magnitude of the momentum of the ball,			**Equation:** $\begin{aligned} p &= m \cdot v \\ &= (0{,}42\ \text{kg})\left(20\,\text{m}\cdot\text{s}^{-1}\right) \\ &= 8{,}4\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1} \end{aligned}$			
Quote the final answer	We quote the answer with the direction of motion included, p = 8,4 kg·m·s^{-1} in the direction of the goal post.						

Exercise:
Momentum of a cricket ball

Problem:

A cricket ball of mass 160 g is bowled at 40 m·s^{-1} towards a batsman. Calculate the momentum of the cricket ball.

Solution:

Identify what information is given and what is asked for	The question explicitly gives	○ the mass of the ball (m = 160 g = 0,16 kg), and ○ the velocity of the ball (v = 40 m·s^{-1})	To calculate the momentum we will use	**Equation:** $p = m \cdot v$	.
Do the	**Equation:**				

calculation

$$\begin{aligned} p &= m \cdot v \\ &= (0,16\ \text{kg})\left(40\,\text{m}\cdot\text{s}^{-1}\right) \\ &= 6,4\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1} \\ &= 6,4\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\,\text{in the direction of the batsman} \end{aligned}$$

Exercise:
Momentum of the Moon

Problem:

The Moon is 384 400 km away from the Earth and orbits the Earth in 27,3 days. If the Moon has a mass of 7,35 x 10^{22}kg, what is the magnitude of its momentum if we assume a circular orbit?

Solution:

Identify what information is given and what is asked for

The question explicitly gives

- the mass of the Moon (m = 7,35 x 10^{22} kg)
- the distance to the Moon (384 400 km = 384 400 000 m = 3,844 x 10^{8} m)
- the time for one orbit of the Moon (27,3 days = 27,3 x 24 x 60 x 60 = 2,36 x 10^{6} s)

We are asked to calculate only the magnitude of the momentum of the Moon (i.e. we do not need to specify a direction). In order to do this we require the mass and the magnitude of the velocity of the Moon, since

Equation:

$$p = m \cdot v$$

Find the magnitude of the velocity of the Moon

The magnitude of the average velocity is the same as the speed. Therefore:

Equation:

$$s = \frac{d}{\Delta t}$$

We are given the time the Moon takes for one orbit but not how far it travels in that time. However, we can work this out from the distance to the Moon and the fact that the Moon has a circular orbit. Using the equation for the circumference, C, of a circle in terms of its radius, we can determine the

Equation:

$$\begin{aligned} C &= 2\pi r \\ &= 2\pi\left(3,844\times 10^{8}\text{m}\right) \\ &= 2,42\times 10^{9}\ \text{m} \end{aligned}$$

Combining the distance travelled by the Moon in an orbit and the time taken by the Moon to complete one orbit, we can determine the magnitude of the Moon's velocity or speed,

Equation:

$$\begin{aligned} s &= \frac{d}{\Delta t} \\ &= \frac{C}{T} \\ &= \frac{2,42\times 10^{9} m}{2,36\times 10^{6} s} \\ &= 1,02\times 10^{3}\ \text{m}\cdot\text{s}^{-} \end{aligned}$$

distance travelled by the Moon in one orbit:

Finally calculate the momentum and quote the answer

The magnitude of the Moon's momentum is:

Equation:

$$\begin{aligned} p &= m \cdot v \\ &= \left(7,35 \times 10^{22}\ \mathrm{kg}\right)\left(1,02 \times 10^{3}\,\mathrm{m \cdot s^{-1}}\right) \\ &= 7,50 \times 10^{25}\,\mathrm{kg \cdot\ m \cdot s^{-1}} \end{aligned}$$

Vector Nature of Momentum

As we have said, momentum is a vector quantity. Since momentum is a vector, the techniques of vector addition discussed in Chapter [link] must be used to calculate the total momentum of a system.

Exercise:

Calculating the Total Momentum of a System

Problem:

Two billiard balls roll towards each other. They each have a mass of 0,3 kg. Ball 1 is moving at $v_1 = 1\,\mathrm{m \cdot s^{-1}}$ to the right, while ball 2 is moving at $v_2 = 0,8\,\mathrm{m \cdot s^{-1}}$ to the left. Calculate the total momentum of the system.

Solution:

Identify what information is given and what is asked for

The question explicitly gives

- the mass of each ball,
- the velocity of ball 1, v_1, and
- the velocity of ball 2, v_2,

all in the correct units!

We are asked to calculate the **total momentum of the system**. In this example our system consists of two balls. To find the total momentum we must determine the momentum of each ball and add them.

Equation:

$$p_{total} = p_1 + p_2$$

Since ball 1 is moving to the right, its momentum is in this direction, while the second ball's momentum is directed towards the left.

Thus, we are required to find the sum of two vectors acting along the same straight line. The algebraic method of vector addition introduced in Chapter [link] can thus be used.

Choose a frame of reference

Let us choose right as the positive direction, then obviously left is negative.

Calculate the momentum

The total momentum of the system is then the sum of the two momenta taking the directions of the velocities into account. Ball 1 is travelling at 1 m·s^{-1}to the right or

Equation:

$$\begin{aligned} p_{total} &= m_1 v_1 + m_2 v_2 \\ &= \left(0,3\ \mathrm{kg}\right)\left(+1\,\mathrm{m \cdot s^{-1}}\right) + \left(0,3\ \mathrm{kg}\right)\left(-0,8\,\mathrm{m \cdot s^{-1}}\right) \\ &= \left(+0,3\,\mathrm{kg \cdot\ m \cdot s^{-1}}\right) + \left(-0,24\,\mathrm{kg \cdot\ m \cdot s^{-1}}\right) \\ &= +0,06\,\mathrm{kg \cdot\ m \cdot s^{-1}} \\ &= 0,06\,\mathrm{kg \cdot\ m \cdot s^{-1}}\ \text{to the right} \end{aligned}$$

In the last step the direction was added in words. Since the result in the second last line is positive, the total

+1 m·s^{-1}. Ball 2 is travelling at 0,8 m·s^{-1}to the left or -0,8 m·s^{-1}. Thus,		momentum of the system is in the positive direction (i.e. to the right).

Exercise

1.
   1. The fastest recorded delivery for a cricket ball is 161,3 km·hr^{-1}, bowled by Shoaib Akhtar of Pakistan during a match against England in the 2003 Cricket World Cup, held in South Africa. Calculate the ball's momentum if it has a mass of 160 g.
   2. The fastest tennis service by a man is 246,2 km·hr^{-1}by Andy Roddick of the United States of America during a match in London in 2004. Calculate the ball's momentum if it has a mass of 58 g.
   3. The fastest server in the women's game is Venus Williams of the United States of America, who recorded a serve of 205 km·hr^{-1}during a match in Switzerland in 1998. Calculate the ball's momentum if it has a mass of 58 g.
   4. If you had a choice of facing Shoaib, Andy or Venus and didn't want to get hurt, who would you choose based on the momentum of each ball.
2. Two golf balls roll towards each other. They each have a mass of 100 g. Ball 1 is moving at v_1 = 2,4 m·s^{-1}to the right, while ball 2 is moving at v_2 = 3 m·s^{-1}to the left. Calculate the total momentum of the system.
3. Two motor cycles are involved in a head on collision. Motorcycle A has a mass of 200 kg and was travelling at 120 km·hr^{-1}south. Motor cycle B has a mass of 250 kg and was travelling north at 100 km·hr^{-1}. A and B are about to collide. Calculate the momentum of the system before the collision takes place.

Change in Momentum

Let us consider a tennis ball (mass = 0,1 kg) that is dropped at an initial velocity of 5 m·s^{-1}and bounces back at a final velocity of 3 m·s^{-1}. As the ball approaches the floor it has a momentum that we call the momentum before the collision. When it moves away from the floor it has a different momentum called the momentum after the collision. The bounce on the floor can be thought of as a collision taking place where the floor exerts a force on the tennis ball to change its momentum.

The momentum before the bounce can be calculated as follows:

Because momentum and velocity are vectors, we have to choose a direction as positive. For this example we choose the initial direction of motion as positive, in other words, downwards is positive.

Equation:

$$\begin{aligned} p_i &= m \cdot v_i \\ &= (0,1\ \text{kg})\left(+5\,\text{m}\cdot\text{s}^{-1}\right) \\ &= 0,5\ \ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\,\text{downwards} \end{aligned}$$

When the tennis ball bounces back it changes direction. The final velocity will thus have a negative value. The momentum after the bounce can be calculated as follows:

Equation:

$$
\begin{aligned}
p_f &= m \cdot v_f \\
&= (0,1\ \mathrm{kg})\left(-3\,\mathrm{m}\cdot\mathrm{s}^{-1}\right) \\
&= -0,3\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} \\
&= 0,3\ \ \mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1}\,\text{upwards}
\end{aligned}
$$

Now let us look at what happens to the momentum of the tennis ball. The momentum changes during this bounce. We can calculate the change in momentum as follows:

Again we have to choose a direction as positive and we will stick to our initial choice as downwards is positive. This means that the final momentum will have a negative number.

Equation:

$$
\begin{aligned}
\Delta p &= p_f - p_i \\
&= m \cdot v_f - m \cdot v_i \\
&= (-0,3\ \mathrm{kg}) - \left(0,5\,\mathrm{m}\cdot\mathrm{s}^{-1}\right) \\
&= -0,8\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} \\
&= 0,8\ \ \mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1}\,\text{upwards}
\end{aligned}
$$

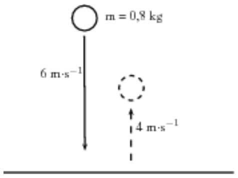
You will notice that this number is bigger than the previous momenta calculated. This is should be the case as the ball needed to be stopped and then given momentum to bounce back.

Exercise:

Change in Momentum

Problem:

A rubber ball of mass 0,8 kg is dropped and strikes the floor with an initial velocity of 6 m·s^{-1}. It bounces back with a final velocity of 4 m·s^{-1}. Calculate the change in the momentum of the rubber ball caused by the floor.



Solution:

					Equation:	
Identify the information given and what is asked	The question explicitly gives	○ the ball's mass (m = 0,8 kg), ○ the ball's initial velocity (v$_i$ = 6 m·s^{-1}), and ○ the ball's final	all in the correct units.	We are asked to calculate the change in momentum of the ball,	$\Delta p = mv_f - mv_i$	We have everything we need to find Δp. Since the initial momentum is directed downwards and the final momentum is in the upward direction, we can use the algebraic method of subtraction discussed in the vectors chapter.

velocity
(v_f = 4
m·s^{-1})

Choose a frame of reference

Let us choose down as the positive direction.

Do the calculation and quote the answer

Equation:

$$\begin{aligned} \Delta p &= mv_f - mv_i \\ &= (0,8\ \text{kg})\left(-4\,\text{m}\cdot\text{s}^{-1}\right) - (0,8\ \text{kg})\left(+6\,\text{m}\cdot\text{s}^{-1}\right) \\ &= \left(-3,2\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\right) - \left(4,8\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\right) \\ &= -8 \\ &= 8\ \ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\ \ \text{upwards} \end{aligned}$$

Exercise

1. Which expression accurately describes the change of momentum of an object?
    1. **A** $\frac{F}{m}$
    2. **B** $\frac{F}{t}$
    3. **C** $F \cdot m$
    4. **D** $F \cdot t$
2. A child drops a ball of mass 100 g. The ball strikes the ground with a velocity of 5 m·s^{-1} and rebounds with a velocity of 4 m·s^{-1}. Calculate the change of momentum of the ball.
3. A 700 kg truck is travelling north at a velocity of 40 km·hr^{-1} when it is approached by a 500 kg car travelling south at a velocity of 100 km·hr^{-1}. Calculate the total momentum of the system.

Newton's Second Law revisited

You have learned about Newton's Second Law of motion earlier in this chapter. Newton's Second Law describes the relationship between the motion of an object and the net force on the object. We said that the motion of an object, and therefore its momentum, can only change when a resultant force is acting on it. We can therefore say that because a net force causes an object to move, it also causes its momentum to change. We can now define Newton's Second Law of motion in terms of momentum.

Newton's Second Law of Motion (N2)

> The net or resultant force acting on an object is equal to the rate of change of momentum.

Mathematically, Newton's Second Law can be stated as:
Equation:

$$F_{net} = \frac{\Delta p}{\Delta t}$$

Impulse

Impulse is the product of the net force and the time interval for which the force acts. Impulse is defined as:
Equation:

$$\text{Impulse} = F \cdot \Delta t$$

However, from Newton's Second Law, we know that
Equation:

$$\begin{aligned} F &= \frac{\Delta p}{\Delta t} \\ \therefore \quad F \cdot \Delta t &= \Delta p \\ &= \text{Impulse} \end{aligned}$$

Therefore,
Equation:

$$\text{Impulse} = \Delta p$$

Impulse is equal to the change in momentum of an object. From this equation we see, that for a given change in momentum, $F_{net}\Delta t$ is fixed. Thus, if F_{net} is reduced, Δt must be increased (i.e. a smaller resultant force must be applied for longer to bring about the same change in momentum). Alternatively if Δt is reduced (i.e. the resultant force is applied for a shorter period) then the resultant force must be increased to bring about the same change in momentum.
Exercise:
Impulse and Change in momentum

Problem:

A 150 N resultant force acts on a 300 kg trailer. Calculate how long it takes this force to change the trailer's velocity from 2 m·s^{-1}to 6 m·s^{-1}in the same direction. Assume that the forces acts to the right.

Solution:

Identify what information is given and what is asked for

The question explicitly gives

- the trailer's mass as 300 kg,
- the trailer's initial velocity as 2 m·s^{-1}to the right,
- the trailer's final velocity as 6 m·s^{-1}to the right, and
- the resultant force acting on the object

all in the correct units!

We are asked to calculate the time taken Δt to accelerate the trailer from the 2 to 6 m·s^{-1}. From the Law of Momentum,

Equation:

$$\begin{aligned} F_{net}\Delta t &= \Delta p \\ &= mv_f - mv_i \\ &= m(v_f - v_i). \end{aligned}$$

Thus we have everything we need to find Δt !

Choose a frame of reference

Choose right as the positive direction.

Do the calculation and quote the final answer

Equation:

$$\begin{aligned} F_{net}\Delta t &= m(v_f - v_i) \\ (+150\,\mathrm{N})\Delta \mathrm{t} &= (300\ \mathrm{kg})\left(\left(+6\,\mathrm{m}\cdot\mathrm{s}^{-1}\right) - \left(+2\,\mathrm{m}\cdot\mathrm{s}^{-1}\right)\right) \\ (+150\,\mathrm{N})\Delta \mathrm{t} &= (300\ \mathrm{kg})\left(+4\,\mathrm{m}\cdot\mathrm{s}^{-1}\right) \\ \Delta t &= \frac{(300\ \mathrm{kg})\left(+4\,\mathrm{m}\cdot\mathrm{s}^{-1}\right)}{+150\,\mathrm{N}} \\ \Delta t &= 8\,\mathrm{s} \end{aligned}$$

It takes 8 s for the force to change the object's velocity from 2 m·s^{-1}to the right to 6 m·s^{-1}to the right.

Exercise:
Impulsive cricketers!

Problem:

A cricket ball weighing 156 g is moving at 54 km·hr^{-1}towards a batsman. It is hit by the batsman back towards the bowler at 36 km·hr^{-1}. Calculate

1. the ball's impulse, and
2. the average force exerted by the bat if the ball is in contact with the bat for 0,13 s.

Solution:

Identify what information is given and what is asked for

The question explicitly gives

- the ball's mass,
- the ball's initial velocity,
- the ball's final velocity, and
- the time of contact between bat and ball

We are asked to calculate the impulse

Equation:

$$\text{Impulse} = \Delta p = F_{net}\Delta t$$

Since we do not have the force exerted by the bat on the ball (F $_{net}$), we have to calculate the impulse from the change in momentum of the ball. Now, since

Equation:

$$\begin{aligned} \Delta p &= p_f - p_i \\ &= mv_f - mv_i, \end{aligned}$$

we need the ball's mass, initial velocity and final velocity which we are given.

Convert to S.I. units

Firstly let us change units for the mass

Equation:

$$\begin{aligned} 1000\ \mathrm{g} &= 1\ \mathrm{kg} \\ \mathrm{So}, 1\ \mathrm{g} &= \frac{1}{1000}\ \mathrm{kg} \\ \therefore 156 \times 1\ \mathrm{g} &= 156 \times \frac{1}{1000}\ \mathrm{kg} \\ &= 0,156\ \ \mathrm{kg} \end{aligned}$$

Next we change units for the velocity

Equation:

$$\begin{aligned} 1\ \mathrm{km}\cdot\mathrm{h}^{-1} &= \frac{1000\,\mathrm{m}}{3\ 600\,\mathrm{s}} \\ \therefore 54 \times 1\ \mathrm{km}\cdot\mathrm{h}^{-1} &= 54 \times \frac{1\,000\,\mathrm{m}}{3\,600\,\mathrm{s}} \\ &= 15\,\mathrm{m}\cdot\mathrm{s}^{-1} \end{aligned}$$

Similarly, 36 km·hr^{-1}= 10 m ·s^{-1}.

Choose a

frame of reference Let us choose the direction from the batsman to the bowler as the positive direction. Then the initial velocity of the ball is v_i = -15 m·s^{-1}, while the final velocity of the ball is v_f = 10 m·s^{-1}.

Calculate the momentum Now we calculate the change in momentum,

Equation:

$$\begin{aligned} p &= p_f - p_i \\ &= mv_f - mv_i \\ &= m(v_f - v_i) \\ &= (0,156\ \text{kg})\left((+10\,\text{m}\cdot\text{s}^{-1}) - (-15\,\text{m}\cdot\text{s}^{-1})\right) \\ &= +3,9\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1} \\ &= 3,9\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\ \text{in the direction from batsman to bowler} \end{aligned}$$

Determine the impulse Finally since impulse is just the change in momentum of the ball,

Equation:

$$\begin{aligned} \text{Impulse} &= \Delta p \\ &= 3,9\,\text{kg}\cdot\ \text{m}\cdot\text{s}^{-1}\ \text{in the direction from the batsman to the bowler} \end{aligned}$$

Determine the average force exerted by the bat **Equation:**

$$\text{Impulse} = F_{net}\Delta t = \Delta p$$

We are given Δt and we have calculated the impulse of the ball.

Equation:

$$\begin{aligned} F_{net}\Delta t &= \text{Impulse} \\ F_{net}\,(0,13\,\text{s}) &= +3,9\,\text{N}\cdot\ \text{s} \\ F_{net} &= \frac{+3,9\,\text{N}\cdot\ \text{s}}{0,13\,\text{s}} \\ &= +30\,\text{N} \\ &= 30\,\text{N} \qquad \text{in the direction from batsman t} \end{aligned}$$

Exercise

1. Which one of the following is NOT a unit of impulse?
    1. **A**$N \cdot s$
    2. **B**$kg \cdot\ \text{m}\cdot\text{s}^{-1}$
    3. **C**$J \cdot\ \text{m}\cdot\text{s}^{-1}$
    4. **D**$J \cdot m^{-1} \cdot s$
2. A toy car of mass 1 kg moves eastwards with a speed of 2 m·s^{-1}. It collides head-on with a toy train. The train has a mass of 2 kg and is moving at a speed of 1,5 m·s^{-1}westwards. The car rebounds (bounces back) at 3,4 m·s^{-1}and the train rebounds at 1,2 m·s^{-1}.
    1. Calculate the change in momentum for each toy.
    2. Determine the impulse for each toy.
    3. Determine the duration of the collision if the magnitude of the force exerted by each toy is 8 N.
3. A bullet of mass 20 g strikes a target at 300 m·s^{-1}and exits at 200 m·s^{-1}. The tip of the bullet takes 0,0001s to pass through the target. Determine:
    1. the change of momentum of the bullet.
    2. the impulse of the bullet.

    3. the magnitude of the force experienced by the bullet.

4. A bullet of mass 20 g strikes a target at 300 $\text{m}\cdot\text{s}^{-1}$. Determine under which circumstances the bullet experiences the greatest change in momentum, and hence impulse:
    1. When the bullet exits the target at 200 $\text{m}\cdot\text{s}^{-1}$.
    2. When the bullet stops in the target.
    3. When the bullet rebounds at 200 $\text{m}\cdot\text{s}^{-1}$.
5. A ball with a mass of 200 g strikes a wall at right angles at a velocity of 12 $\text{m}\cdot\text{s}^{-1}$and rebounds at a velocity of 9 $\text{m}\cdot\text{s}^{-1}$.
    1. Calculate the change in the momentum of the ball.
    2. What is the impulse of the wall on the ball?
    3. Calculate the magnitude of the force exerted by the wall on the ball if the collision takes 0,02s.
6. If the ball in the previous problem is replaced with a piece of clay of 200 g which is thrown against the wall with the same velocity, but then sticks to the wall, calculate:
    1. The impulse of the clay on the wall.
    2. The force exerted by the clay on the wall if it is in contact with the wall for 0,5 s before it comes to rest.

Conservation of Momentum

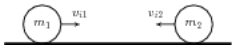
In the absence of an external force acting on a system, momentum is conserved.

Conservation of Linear Momentum

> The total linear momentum of an isolated system is constant. An isolated system has no forces acting on it from the outside.

This means that in an isolated system the total momentum before a collision or explosion is equal to the total momentum after the collision or explosion.

Consider a simple collision of two billiard balls. The balls are rolling on a frictionless surface and the system is isolated. So, we can apply conservation of momentum. The first ball has a mass m_1 and an initial velocity v_{i1}. The second ball has a mass m_2 and moves towards the first ball with an initial velocity v_{i2}. This situation is shown in [link].

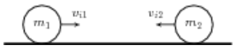


Before the collision.

The total momentum of the system before the collision, p_i is:
Equation:

$$p_i = m_1 v_{i1} + m_2 v_{i2}$$

After the two balls collide and move away they each have a different momentum. If the first ball has a final velocity of v_{f1} and the second ball has a final velocity of v_{f2} then we have the situation shown in [link].



After the collision.

The total momentum of the system after the collision, p_f is:

Equation:

$$p_f = m_1 v_{f1} + m_2 v_{f2}$$

This system of two balls is isolated since there are no external forces acting on the balls. Therefore, by the principle of conservation of linear momentum, the total momentum before the collision is equal to the total momentum after the collision. This gives the equation for the conservation of momentum in a collision of two objects,

Equation:

$$p_i = p_f$$

Equation:

$$m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2}$$

where: m_1 is the mass of object 1 (kg)
m_2 is the mass of object 2 (kg)
v_{i1} is the initial velocity of object 1 (m·s^{-1}+ direction)
v_{i2} is the initial velocity of object 2 (m·s^{-1}- direction)
v_{f1} is the final velocity of object 1 (m·s^{-1}- direction)
v_{f2} is the final velocity of object 2 (m·s^{-1}+ direction)

This equation is always true - momentum is always conserved in collisions.

Exercise:

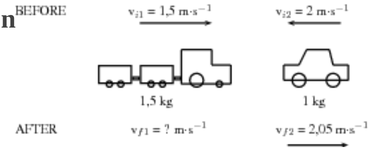
Conservation of Momentum 1

Problem:

A toy car of mass 1 kg moves westwards with a speed of 2 m·s^{-1}. It collides head-on with a toy train. The train has a mass of 1,5 kg and is moving at a speed of 1,5 m·s^{-1}eastwards. If the car rebounds at 2,05 m·s^{-1}, calculate the velocity of the train.

Solution:

Draw rough sketch of the situation



Choose a frame of reference

We will choose to the east as positive.

Apply the Law of Conservation of momentum

Equation:

$$
\begin{aligned}
p_i &= p_f \\
m_1 v_{i1} + m_2 v_{i2} &= m_1 v_{f1} + m_2 v_{f2} \\
(1,5\ \mathrm{kg})\big(+1,5\,\mathrm{m}\cdot\mathrm{s}^{-1}\big) + (2\ \mathrm{kg})\big(-2\,\mathrm{m}\cdot\mathrm{s}^{-1}\big) &= (1,5\ \mathrm{kg})\,(v_{f1}) + (2\ \mathrm{kg})\ \big(2,05\,\mathrm{m}\cdot\mathrm{s} \\
2,25\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} - 4\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} - 4,1\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} &= (1,5\ \mathrm{kg})\ v_{f1} \\
5,85\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} &= (1,5\ \mathrm{kg})\ v_{f1} \\
v_{f1} &= 3,9\,\mathrm{m}\cdot\mathrm{s}^{-1}\,\mathrm{eastwards}
\end{aligned}
$$

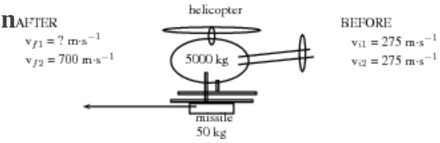
Exercise:
Conservation of Momentum 2

Problem:

A helicopter flies at a speed of 275 m·s^{-1}. The pilot fires a missile forward out of a gun barrel at a speed of 700 m·s^{-1}. The respective masses of the helicopter and the missile are 5000 kg and 50 kg. Calculate the new speed of the helicopter immmediately after the missile had been fired.

Solution:

Draw rough sketch of the situation



AFTER
v_{f1} = ? m·s^{-1}
v_{f2} = 700 m·s^{-1}
helicopter
5000 kg
missile
50 kg
BEFORE
v_{i1} = 275 m·s^{-1}
v_{i2} = 275 m·s^{-1}

helicopter and missile

Analyse the question and list what is given	m_1 = 5000 kg	m_2 = 50 kg	v_{i1} = v_{i2} = 275 m·s^{-1}	v_{f1} = ?	v_{f2} = 700 m·s^{-1}

Apply the Law of Conservation of momentum

The helicopter and missile are connected initially and move at the same velocity. We will therefore combine their masses and change the momentum equation as follows:

Equation:

$$
\begin{aligned}
p_i &= p_f \\
(m_1 + m_2) v_i &= m_1 v_{f1} + m_2 v_{f2} \\
(5000\,\mathrm{kg} + 50\ \mathrm{kg})\big(275\,\mathrm{m}\cdot\mathrm{s}^{-1}\big) &= (5000\ \mathrm{kg})\,(v_{f1}) + (50\ \mathrm{kg})\ \big(700\,\mathrm{m}\ \cdot \\
1388750\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} - 35000\,\mathrm{kg}\cdot\ \mathrm{m}\cdot\mathrm{s}^{-1} &= (5000\ \mathrm{kg})\,(v_{f1}) \\
v_{f1} &= 270,75\,\mathrm{m}\cdot\mathrm{s}^{-1}
\end{aligned}
$$

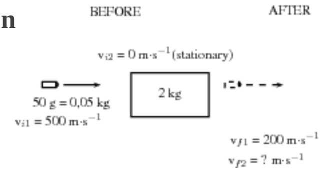
Exercise:
Conservation of Momentum 3

Problem:

A bullet of mass 50 g travelling horizontally at 500 m·s^{-1}strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet goes through the block and comes out on the other side at 200 m·s^{-1}. Calculate the speed of the block after the bullet has come out the other side.

Solution:

Draw rough sketch of the situation


BEFORE
AFTER
v_{i2} = 0 m·s^{-1} (stationary)
2 kg
50 g = 0,05 kg
v_{i1} = 500 m·s^{-1}
v_{f1} = 200 m·s^{-1}
v_{f2} = ? m·s^{-1}

Choose a frame of reference

We will choose to the right as positive.

Apply the Law of Conservation of momentum

Equation:

$$\begin{aligned} p_i &= p_f \\ m_1 v_{i1} + m_2 v_{i2} &= m_1 v_{f1} + m_2 v_{f2} \\ (0,05\ \text{kg})\left(+500\,\text{m}\cdot\text{s}^{-1}\right) + (2\ \text{kg})\left(0\,\text{m}\cdot\text{s}^{-1}\right) &= (0,05\ \text{kg})\left(+200\,\text{m}\cdot\text{s}^{-1}\right) + (2\ \text{kg})\left(v_{f2}\right) \\ 25 + 0 - 10 &= 2\ v_{f2} \\ v_{f2} &= 7,5\,\text{m}\cdot\text{s}^{-1}\ \text{in the same direction as the} \end{aligned}$$

Physics in Action: Impulse

A very important application of impulse is improving safety and reducing injuries. In many cases, an object needs to be brought to rest from a certain initial velocity. This means there is a certain specified change in momentum. If the time during which the momentum changes can be increased then the force that must be applied will be less and so it will cause less damage. This is the principle behind arrestor beds for trucks, airbags, and bending your knees when you jump off a chair and land on the ground.

Air-Bags in Motor Vehicles

Air bags are used in motor vehicles because they are able to reduce the effect of the force experienced by a person during an accident. Air bags extend the time required to stop the momentum of the driver and passenger. During a collision, the motion of the driver and passenger carries them towards the windshield. If they are stopped by a collision with the windshield, it would result in a large force exerted over a short time in order to bring them to a stop. If instead of hitting the windshield, the driver and passenger hit an air bag, then the time of the impact is increased. Increasing the time of the impact results in a decrease in the force.

Padding as Protection During Sports

The same principle explains why wicket keepers in cricket use padded gloves or why there are padded mats in gymnastics. In cricket, when the wicket keeper catches the ball, the padding is slightly compressible, thus reducing the effect of the force on the wicket keepers hands. Similarly, if a gymnast falls, the padding compresses and reduces the effect of the force on the gymnast's body.

Arrestor Beds for Trucks

An arrestor bed is a patch of ground that is softer than the road. Trucks use these when they have to make an emergency stop. When a trucks reaches an arrestor bed the time interval over which the momentum is changed is increased. This decreases the force and causes the truck to slow down.

Follow-Through in Sports

In sports where rackets and bats are used, like tennis, cricket, squash, badminton and baseball, the hitter is often encouraged to follow-through when striking the ball. High speed films of the collisions between bats/rackets and balls have shown that following through increases the time over which the collision between the racket/bat and ball occurs. This increase in the time of the collision causes an increase in the velocity change of the ball. This means that a hitter can cause the ball to leave the racket/bat faster by following through. In these sports, returning the ball with a higher velocity often increases the chances of success.

Crumple Zones in Cars

Another safety application of trying to reduce the force experienced is in crumple zones in cars. When two cars have a collision, two things can happen:

1. the cars bounce off each other, or
2. the cars crumple together.

Which situation is more dangerous for the occupants of the cars? When cars bounce off each other, or rebound, there is a larger change in momentum and therefore a larger impulse. A larger impulse means that a greater force is experienced by the occupants of the cars. When cars crumple together, there is a smaller change in momentum and therefore a smaller impulse. The smaller impulse means that the occupants of the cars experience a smaller force. Car manufacturers use this idea and design crumple zones into cars, such that the car has a greater chance of crumpling than rebounding in a collision. Also, when the car crumples, the change in the car's momentum happens over a longer time. Both these effects result in a smaller force on the occupants of the car, thereby increasing their chances of survival.

Egg Throw

This activity demonstrates the effect of impulse and how it is used to improve safety. Have two learners hold up a bed sheet or large piece of fabric. Then toss an egg at the sheet. The egg should not break, because the collision between the egg and the bed sheet lasts over an extended period of time since the bed sheet has some give in it. By increasing the time of the collision, the force of the impact is minimized. Take care to aim at the sheet, because if you miss the sheet, you will definitely break the egg and have to clean up the mess!

Exercise

1. A canon, mass 500 kg, fires a shell, mass 1 kg, horizontally to the right at 500 $\text{m}\cdot\text{s}^{-1}$. What is the magnitude and direction of the initial recoil velocity of the canon?
2. A trolley of mass 1 kg is moving with a speed of 3 $\text{m}\cdot\text{s}^{-1}$. A block of wood, mass 0,5 kg, is dropped vertically into the trolley. Immediately after the collision, the speed of the trolley and block is 2 $\text{m}\cdot\text{s}^{-1}$. By way of calculation, show whether momentum is conserved in the collision.
3. A 7200 kg empty railway truck is stationary. A fertilizer firm loads 10800 kg fertilizer into the truck. A second, identical, empty truck is moving at 10 $\text{m}\cdot\text{s}^{-1}$when it collides with the loaded truck.
    1. If the empty truck stops completely immediately after the collision, use a conservation law to calculate the velocity of the loaded truck immediately after the collision.

    2. Calculate the distance that the loaded truck moves after collision, if a constant frictional force of 24 kN acts on the truck.

4. A child drops a squash ball of mass 0,05 kg. The ball strikes the ground with a velocity of 4 m·s^{-1}and rebounds with a velocity of 3 m·s^{-1}. Does the law of conservation of momentum apply to this situation? Explain.
5. A bullet of mass 50 g travelling horizontally at 600 m·s^{-1}strikes a stationary wooden block of mass 2 kg resting on a smooth horizontal surface. The bullet gets stuck in the block.
    1. Name and state the principle which can be applied to find the speed of the block-and-bullet system after the bullet entered the block.
    2. Calculate the speed of the bullet-and-block system immediately after impact.
    3. If the time of impact was 5 x 10^{-4} seconds, calculate the force that the bullet exerts on the block during impact.

Torque and Levers

Torque

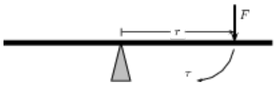
This chapter has dealt with forces and how they lead to motion in a straight line. In this section, we examine how forces lead to rotational motion.

When an object is fixed or supported at one point and a force acts on it a distance away from the support, it tends to make the object turn. The moment of force or *torque* (symbol, τ read *tau*) is defined as the product of the distance from the support or pivot (r) and the component of force perpendicular to the object, $F_{\perp}$.

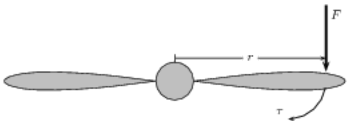
Equation:

$$\tau = F_{\perp} \cdot r$$

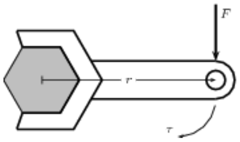
Torque can be seen as a rotational force. The unit of torque is N·m and torque is a vector quantity. Some examples of where torque arises are shown in Figures [link], [link] and [link].



The force exerted on one side of a see-saw causes it to swing.



The force exerted on the edge of a propellor causes the propellor to spin.



The force exerted on a spanner helps to loosen the bolt.

For example in [link], if a force F of 10 N is applied perpendicularly to the spanner at a distance r of 0,3 m from the center of the bolt, then the torque applied to the bolt is:

Equation:

$$\begin{aligned}\tau &= F_{\perp} \cdot r \\ &= (10\ \mathrm{N})(0,3\,\mathrm{m}) \\ &= 3\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

If the force of 10 N is now applied at a distance of 0,15 m from the centre of the bolt, then the torque is:

Equation:

$$\begin{aligned}\tau &= F_{\perp} \cdot r \\ &= (10\ \mathrm{N})(0,15\,\mathrm{m}) \\ &= 1,5\,\mathrm{N}\cdot\mathrm{m}\end{aligned}$$

This shows that there is less torque when the force is applied closer to the bolt than further away.

Note: Loosening a bolt

If you are trying to loosen (or tighten) a bolt, apply the force on the spanner further away from the bolt, as this results in a greater torque to the bolt making it easier to loosen.

Note: Any component of a force exerted parallel to an object will not cause the object to turn. Only perpendicular components cause turning.

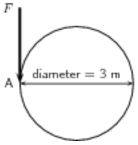
Note: Torques

The direction of a torque is either clockwise or anticlockwise. When torques are added, choose one direction as positive and the opposite direction as negative. If equal clockwise and anticlockwise torques are applied to an object, they will cancel out and there will be no net turning effect.

Exercise:
Merry-go-round

Problem:

Several children are playing in the park. One child pushes the merry-go-round with a force of 50 N. The diameter of the merry-go-round is 3,0 m. What torque does the child apply if the force is applied perpendicularly at point A?



Solution:

Identify what has been given

The following has been given:

- the force applied, F = 50 N
- the diameter of the merry-go-round, $2r$ = 3 m, therefore r = 1,5 m.

The quantities are in SI units.

Decide how to approach the problem

We are required to determine the torque applied to the merry-go-round. We can do this by using:

Equation:

$$\tau = F_{\perp} \cdot r$$

We are given $F_{\perp}$ and we are given the diameter of the merry-go-round. Therefore, r = 1,5 m.

Solve the problem

Equation:

$$\begin{aligned} \tau &= F_{\perp} \cdot r \\ &= (50\,\mathrm{N})(1,5\,\mathrm{m}) \\ &= 75\,\mathrm{N} \cdot \mathrm{m} \end{aligned}$$

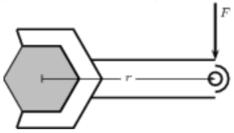
Write the final answer

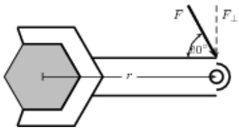
75 $\mathrm{N} \cdot \mathrm{m}$ of torque is applied to the merry-go-round.

Exercise:
Flat tyre

Problem:

Kevin is helping his dad replace the flat tyre on the car. Kevin has been asked to undo all the wheel nuts. Kevin holds the spanner at the same distance for all nuts, but applies the force at two angles (90° and 60°). If Kevin applies a force of 60 N, at a distance of 0,3 m away from the nut, which angle is the best to use? Prove your answer by means of calculations.





Solution:

Identify what has been given

The following has been given:

- the force applied, F = 60 N
- the angles at which the force is applied, $\theta = 90^{\circ}$ and $\theta = 60^{\circ}$

The quantities are in SI units.

- the distance from the centre of the nut at which the force is applied, r = 0,3 m

Decide how to approach the problem

We are required to determine which angle is more better to use. This means that we must find which angle gives the higher torque. We can use

Equation:

$$\tau = F_\perp \cdot r$$

to determine the torque. We are given F for each situation. $F_\perp = F$ sin θ and we are given θ. We are also given the distance away from the nut, at which the force is applied.

Solve the problem for $F_\perp = F$

Equation:

$$\begin{aligned} \tau &= F_\perp \cdot r \\ &= (60\,\mathrm{N})(0,3\,\mathrm{m}) \\ &= 18\,\mathrm{N}\cdot\mathrm{m} \end{aligned}$$

Solve the problem for

Equation:

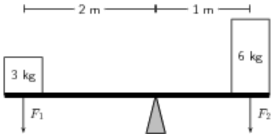
$$\begin{aligned} \tau &= F_\perp \cdot r \\ &= F\ \mathrm{sin}\ \theta \cdot r \\ &= (60\,\mathrm{N})\ \mathrm{sin}\ (\theta)(0,3\,\mathrm{m}) \\ &= 15,6\,\mathrm{N}\cdot\mathrm{m} \end{aligned}$$

Write the final answer

The torque from the perpendicular force is greater than the torque from the force applied at 60°. Therefore, the best angle is 90°.

Mechanical Advantage and Levers

We can use our knowlegde about the moments of forces (torque) to determine whether situations are balanced. For example two mass pieces are placed on a seesaw as shown in [link]. The one mass is 3 kg and the other is 6 kg. The masses are placed at distances of 2 m and 1 m (respectively) from the pivot. By looking at the clockwise and anti-clockwise moments, we can determine whether the seesaw will pivot (move) or not. If the sum of the clockwise and anti-clockwise moments is zero, the seesaw is in equilibrium (i.e. balanced).



The moments of force are balanced.

The clockwise moment can be calculated as follows:

Equation:

$$\begin{aligned} \tau_1 &= F_\perp \cdot r \\ \tau_1 &= (6\ \mathrm{kg})\left(9,8\,\mathrm{m}\cdot\mathrm{s}^{-2}\right)(1m) \\ \tau_1 &= 58,8N\cdot m\ \ \mathrm{clockwise} \end{aligned}$$

The anti-clockwise moment can be calculated as follows:

Equation:

$$\begin{aligned}\tau_2 &= F_{\perp} \cdot r \\ \tau_2 &= (3\ \mathrm{kg})\left(9,8\,\mathrm{m}\cdot\mathrm{s}^{-2}\right)(2m) \\ \tau_2 &= 58,8N \cdot m\ \text{anti}-\text{clockwise}\end{aligned}$$

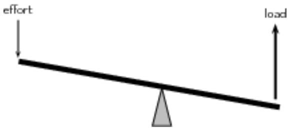
The sum of the moments of force will be zero:

The resultant moment is zero as the clockwise and anti-clockwise moments of force are in opposite directions and therefore cancel each other.

As we see in [link], we can use different distances away from a pivot to balance two different forces. This principle is applied to a lever to make lifting a heavy object much easier.

Lever

> A lever is a rigid object that is used with an appropriate fulcrum or pivot point to multiply the mechanical force that can be applied to another object.



A lever is used to put in a small effort to get out a large load.

Note:
Interesting Fact
Archimedes reputedly said: *Give me a lever long enough and a fulcrum on which to place it, and I shall move the world.*

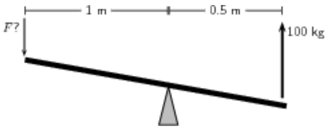
The concept of getting out more than the effort is termed mechanical advantage, and is one example of the principle of moments. The lever allows one to apply a smaller force over a greater distance. For instance to lift a certain unit of weight with a lever with an effort of half a unit we need a distance from the fulcrum in the effort's side to be twice the distance of the weight's side. It also means that to lift the weight 1 meter we need to push the lever for 2 meters. The amount of work done is always the same and independent of the dimensions of the lever (in an ideal lever). The lever only allows to trade force for distance.

Ideally, this means that the mechanical advantage of a system is the ratio of the force that performs the work (output or load) to the applied force (input or effort), assuming there is no friction in the system. In reality, the mechanical advantage will be less than the ideal value by an amount determined by the amount of friction.

Equation:

$$\text{mechanical advantage} = \frac{\text{load}}{\text{effort}}$$

For example, you want to raise an object of mass 100 kg. If the pivot is placed as shown in [link], what is the mechanical advantage of the lever?



A lever is used to put in a small effort to get out a large load.

In order to calculate mechanical advantage, we need to determine the load and effort.

Note: Effort is the input force and load is the output force.

The load is easy to calculate, it is simply the weight of the 100 kg object.
Equation:

$$F_{load} = m \cdot g = 100\,\text{kg} \cdot 9,8\,\text{m} \cdot \text{s}^{-2} = 980\,\text{N}$$

The effort is found by balancing torques.
Equation:

$$\begin{aligned} F_{load} \cdot r_{load} &= F_{effort} \cdot r_{effort} \\ 980\,\text{N} \cdot 0.5\,\text{m} &= F_{effort} \cdot 1\,\text{m} \\ F_{effort} &= \frac{980\,\text{N} \cdot 0.5\,\text{m}}{1\,\text{m}} \\ &= 490\,\text{N} \end{aligned}$$

The mechanical advantage is:
Equation:

$$\begin{aligned} \text{mechanical advantage} &= \frac{\text{load}}{\text{effort}} \\ &= \frac{980\,\text{N}}{490\,\text{N}} \\ &= 2 \end{aligned}$$

Since mechanical advantage is a ratio, it does not have any units.

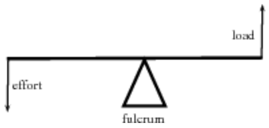
Pulleys

Pulleys change the direction of a tension force on a flexible material, e.g. a rope or cable. In addition, pulleys can be “added together” to create mechanical advantage, by having the flexible material looped over several pulleys in turn. More loops and pulleys increases the mechanical advantage.

Classes of levers

Class 1 levers

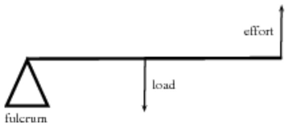
In a class 1 lever the fulcrum is between the effort and the load. Examples of class 1 levers are the seesaw, crowbar and equal-arm balance. The mechanical advantage of a class 1 lever can be increased by moving the fulcrum closer to the load.



Class 1 levers

Class 2 levers

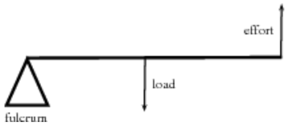
In class 2 levers the fulcrum is at the one end of the bar, with the load closer to the fulcrum and the effort on the other end of bar. The mechanical advantage of this type of lever can be increased by increasing the length of the bar. A bottle opener or wheel barrow are examples of class 2 levers.



Class 2 levers

Class 3 levers

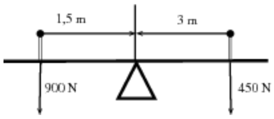
In class 3 levers the fulcrum is also at the end of the bar, but the effort is between the fulcrum and the load. An example of this type of lever is the human arm.



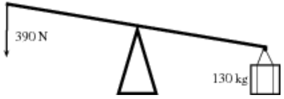
Class 3 levers

Exercise

1. Riyaad applies a force of 120 N on a spanner to undo a nut.
    1. Calculate the moment of the force if he applies the force 0,15 m from the bolt.
    2. The nut does not turn, so Riyaad moves his hand to the end of the spanner and applies the same force 0,2 m away from the bolt. Now the nut begins to move. Calculate the force. Is it bigger or smaller than before?
    3. Once the nuts starts to turn, the moment needed to turn it is less than it was to start it turning. It is now 20 N·m. Calculate the new moment of force that Riyaad now needs to apply 0,2 m away from the nut.
2. Calculate the clockwise and anticlockwise moments of force in the figure below to see if the see-saw is balanced.

3. Jeffrey uses a force of 390 N to lift a load of 130 kg.

    1. Calculate the mechanical advantage of the lever that he is using.
    2. What type of lever is he using? Give a reason for your answer.
    3. If the force is applied 1 m from the pivot, calculate the maximum distance between the pivot and the load.
4. A crowbar is used to lift a box of weight 400 N. The box is placed 75 cm from the pivot. A crow bar is a class 1 lever.
    1. Why is a crowbar a class 1 lever? Draw a diagram to explain your answer.
    2. What force F needs to be applied at a distance of 1,25 m from the pivot to balance the crowbar?
    3. If force F was applied at a distance of 2 m, what would the magnitude of F be?
5. A wheelbarrow is used to carry a load of 200 N. The load is 40 cm from the pivot and the force F is applied at a distance of 1,2 m from the pivot.
    1. What type of lever is a wheelbarrow?
    2. Calculate the force F that needs to be applied to lift the load.
6. The bolts holding a car wheel in place is tightened to a torque of 90 N · m. The mechanic has two spanners to undo the bolts, one with a length of 20 cm and one with a length of 30 cm. Which spanner should he use? Give a reason for your answer by showing calculations and explaining them.

[missing_resource: http://static.slidesharecdn.com/swf/ssplayer2.swf?doc=forces-100510081023-phpapp01&stripped_title=forces-4037441&userName=kwarne]

Summary

- **Newton's First Law**Every object will remain at rest or in uniform motion in a straight line unless it is made to change its state by the action of an *unbalanced force*.
- **Newton's Second Law**The resultant force acting on a body will cause the body to accelerate in the direction of the resultant force The acceleration of the body is directly proportional to the magnitude of the resultant force and inversely proportional to the mass of the object.

- **Newton's Third Law**If body A exerts a force on body B then body B will exert an equal but opposite force on body A.
- **Newton's Law of Universal Gravitation**Every body in the universe exerts a force on every other body. The force is directly proportional to the product of the masses of the bodies and inversely proportional to the square of the distance between them.
- **Equilibrium**Objects at rest or moving with constant velocity are in *equilibrium* and have a *zeroresultant force*.
- **Equilibrant**The *equilibrant* of any number of forces is the single force required to produce equilibrium.
- **Triangle Law for Forces in Equilibrium**Three forces in equilibrium can be represented in magnitude and direction by the three sides of a triangle taken in order.
- **Momentum**The *momentum* of an object is defined as its mass multiplied by its velocity.
- **Momentum of a System**The *total momentum of a system* is the sum of the momenta of each of the objects in the system.
- **Principle of Conservation of Linear Momentum:** $The \rightarrow tall \in earmomentumofanisolatedsystemiscons\tan t'$ or In an isolated system the total momentum before a collision (or explosion) is equal to the total momentum after the collision (or explosion)'.
- **Law of Momentum:**The applied resultant force acting on an object is equal to the rate of change of the object's momentum and this force is in the direction of the change in momentum.

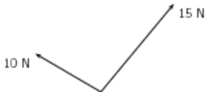
End of Chapter exercises

Forces and Newton's Laws

1. [SC 2003/11] A constant, resultant force acts on a body which can move freely in a straight line. Which physical quantity will remain constant?

   1. acceleration
   2. velocity
   3. momentum
   4. kinetic energy

2. [SC 2005/11 SG1] Two forces, 10 N and 15 N, act at an angle at the same point.

   Which of the following **cannot** be the resultant of these two forces?

   1. **A**2 N
   2. **B**5 N
   3. **C**8 N
   4. **D**20 N

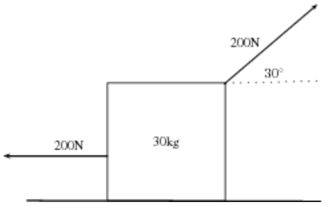
3. A concrete block weighing 250 N is at rest on an inclined surface at an angle of 20°. The magnitude of the normal force, in newtons, is

   - **A**250
   - **B**250 cos 20°
   - **C**250 sin 20°
   - **D**2500 cos 20°

4. A 30 kg box sits on a flat frictionless surface. Two forces of 200 N each are applied to the box as shown in the diagram. Which statement best describes the motion of the box?

   1. **A**The box is lifted off the surface.
   2. **B**The box moves to the right.
   3. **C**The box does not move.

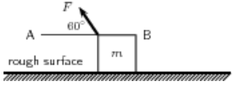
    4. **D**The box moves to the left.



5. A concrete block weighing 200 N is at rest on an inclined surface at an angle of 20°. The normal reaction, in newtons, is

    - ○ **A**200
    - ○ **B**200 cos 20°
    - ○ **C**200 sin 20°
    - ○ **D**2000 cos 20°

6. [SC 2003/11]A box, mass m, is at rest on a rough horizontal surface. A force of constant magnitude F is then applied on the box at an angle of $60^\circ$to the horizontal, as shown.



    If the box has a uniform horizontal acceleration of magnitude, a, the frictional force acting on the box is ...

    1. **A**F cos $60^\circ - ma$ in the direction of A
    2. **B**F cos $60^\circ - ma$ in the direction of B
    3. **C**F sin $60^\circ - ma$ in the direction of A
    4. **D**F sin $60^\circ - ma$ in the direction of B

7. [SC 2002/11 SG] Thabo stands in a train carriage which is moving eastwards. The train suddenly brakes. Thabo continues to move eastwards due to the effect of

    1. **A**his inertia.
    2. **B**the inertia of the train.
    3. **C**the braking force on him.
    4. **D**a resultant force acting on him.

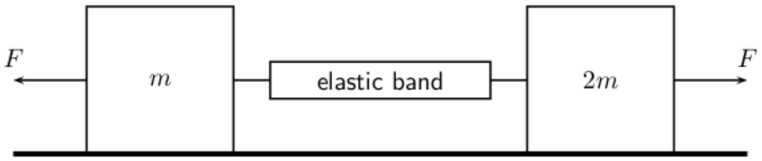
8. [SC 2002/11 HG1] A body slides down a frictionless inclined plane. Which one of the following physical quantities will remain constant throughout the motion?

    1. **A**velocity
    2. **B**momentum
    3. **C**acceleration
    4. **D**kinetic energy

9. [SC 2002/11 HG1] A body moving at a *CONSTANT VELOCITY* on a horizontal plane, has a number of unequal forces acting on it. Which one of the following statements is TRUE?

    1. **A**At least two of the forces must be acting in the same direction.
    2. **B**The resultant of the forces is zero.
    3. **C**Friction between the body and the plane causes a resultant force.
    4. **D**The vector sum of the forces causes a resultant force which acts in the direction of motion.

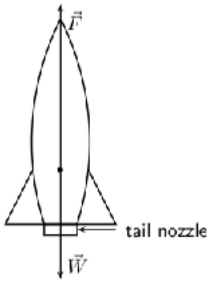
10. [IEB 2005/11 HG] Two masses of m and $2m$ respectively are connected by an elastic band on a frictionless surface. The masses are pulled in opposite directions by two forces each of magnitude F, stretching the elastic band and holding the masses stationary.



Which of the following gives the magnitude of the tension in the elastic band?

1. **A**zero
2. **B**$\frac{1}{2}F$
3. **C**F
4. **D**$2F$

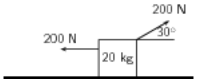
11. [IEB 2005/11 HG] A rocket takes off from its launching pad, accelerating up into the air.

   The rocket accelerates because the magnitude of the upward force, F is greater than the magnitude of the rocket's weight, W. Which of the following statements **best** describes how force F arises?

   1. **A**F is the force of the air acting on the base of the rocket.
   2. **B**F is the force of the rocket's gas jet *pushing down* on the air.
   3. **C**F is the force of the rocket's gas jet *pushing down* on the ground.
   4. **D**F is the reaction to the force that the rocket exerts on the gases which escape out through the tail nozzle.

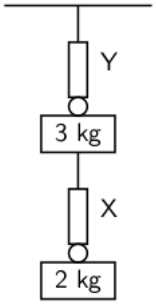
12. [SC 2001/11 HG1] A box of mass 20 kg rests on a smooth horizontal surface. What will happen to the box if two forces each of magnitude 200 N are applied simultaneously to the box as shown in the diagram.

   The box will ...

   1. **A**be lifted off the surface.
   2. **B**move to the left.
   3. **C**move to the right.
   4. **D**remain at rest.

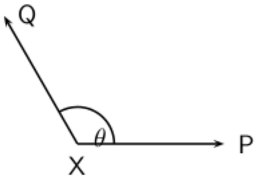
13. [SC 2001/11 HG1] A 2 kg mass is suspended from spring balance X, while a 3 kg mass is suspended from spring balance Y. Balance X is in turn suspended from the 3 kg mass. Ignore the weights of the two spring balances.



The readings (in N) on balances X and Y are as follows:

	X	**Y**
(A)	20	30
(B)	20	50
(C)	25	25
(D)	50	50

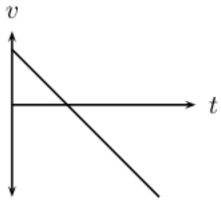
14. [SC 2002/03 HG1] P and Q are two forces of equal magnitude applied simultaneously to a body at X.



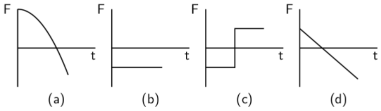
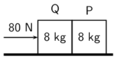
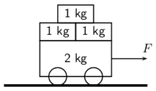
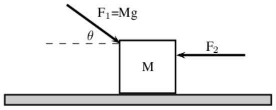
As the angle θ between the forces is **decreased** from 180° to 0°, the magnitude of the resultant of the two forces will

1. **A**initially increase and then decrease.
2. **B**initially decrease and then increase.
3. **C**increase only.
4. **D**decrease only.

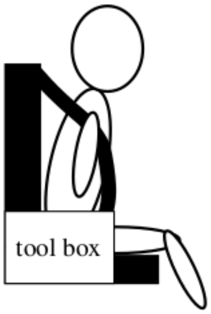
15. [SC 2002/03 HG1] The graph below shows the velocity-time graph for a moving object:



Which of the following graphs could best represent the relationship between the resultant force applied to the object and time?

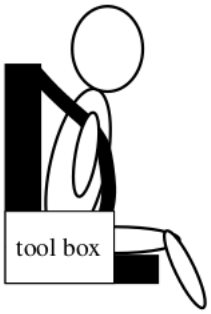
			
(a)	(b)	(c)	(d)

16. [SC 2002/03 HG1] Two blocks each of mass 8 kg are in contact with each other and are accelerated along a frictionless surface by a force of 80 N as shown in the diagram. The force which block Q will exert on block P is equal to ...



1. **A**0 N
2. **B**40 N
3. **C**60 N
4. **D**80 N

17. [SC 2002/03 HG1] Three 1 kg mass pieces are placed on top of a 2 kg trolley. When a force of magnitude F is applied to the trolley, it experiences an acceleration a.



If one of the 1 kg mass pieces falls off while F is still being applied, the trolley will accelerate at ...

1. **A**$\frac{1}{5}a$
2. **B**$\frac{4}{5}a$
3. **C**$\frac{5}{4}a$
4. **D**$5a$

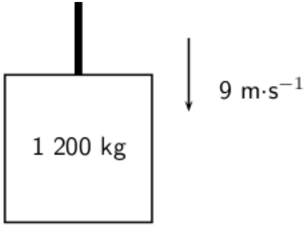
18. [IEB 2004/11 HG1] A car moves along a horizontal road at constant velocity. Which of the following statements is true?

1. **A**The car is not in equilibrium.
2. **B**There are no forces acting on the car.
3. **C**There is zero resultant force.
4. **D**There is no frictional force.

19. [IEB 2004/11 HG1] A crane lifts a load vertically upwards at constant speed. The upward force exerted on the load is F. Which of the following statements is correct?

    1. **A**The acceleration of the load is 9,8 m.s^{-2} downwards.
    2. **B**The resultant force on the load is F.
    3. **C**The load has a weight equal in magnitude to F.
    4. **D**The forces of the crane on the load, and the weight of the load, are an example of a Newton's third law 'action-reaction' pair.

20. [IEB 2004/11 HG1] A body of mass M is at rest on a smooth horizontal surface with two forces applied to it as in the diagram below. Force F_1 is equal to Mg. The force F_1 is applied to the right at an angle θ to the horizontal, and a force of F_2 is applied horizontally to the left.

    How is the body affected when the angle θ is increased?

    1. **A**It remains at rest.
    2. **B**It lifts up off the surface, and accelerates towards the right.
    3. **C**It lifts up off the surface, and accelerates towards the left.
    4. **D**It accelerates to the left, moving along the smooth horizontal surface.

21. [IEB 2003/11 HG1] Which of the following statements correctly explains why a passenger in a car, who is not restrained by the seat belt, continues to move forward when the brakes are applied suddenly?

    1. **A**The braking force applied to the car exerts an equal and opposite force on the passenger.
    2. **B**A forward force (called inertia) acts on the passenger.
    3. **C**A resultant forward force acts on the passenger.
    4. **D**A zero resultant force acts on the passenger.

22. [IEB 2004/11 HG1] A rocket (mass 20 000 kg) accelerates from rest to 40 m·s^{-1}in the first 1,6 seconds of its journey upwards into space. The rocket's propulsion system consists of exhaust gases, which are pushed out of an outlet at its base.

    1. Explain, with reference to the appropriate law of Newton, how the escaping exhaust gases exert an upwards force (thrust) on the rocket.
    2. What is the magnitude of the total thrust exerted on the rocket during the first 1,6 s?
    3. An astronaut of mass 80 kg is carried in the space capsule. Determine the resultant force acting on him during the first 1,6 s.
    4. Explain why the astronaut, seated in his chair, feels “heavier” while the rocket is launched.

23. [IEB 2003/11 HG1 - Sports Car]

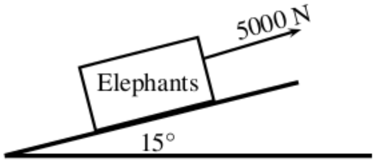
    1. State Newton's Second Law of Motion.
    2. A sports car (mass 1 000 kg) is able to accelerate uniformly from rest to 30 m·s^{-1}in a minimum time of 6 s.

        1. Calculate the magnitude of the acceleration of the car.
        2. What is the magnitude of the resultant force acting on the car during these 6 s?

    3. The magnitude of the force that the wheels of the vehicle exert on the road surface as it accelerates is 7500 N. What is the magnitude of the retarding forces acting on this car?

   4. By reference to a suitable Law of Motion, explain why a headrest is important in a car with such a rapid acceleration.

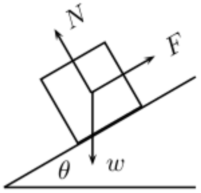
24. [IEB 2005/11 HG1] A child (mass 18 kg) is strapped in his car seat as the car moves to the right at constant velocity along a straight level road. A tool box rests on the seat beside him.

    The driver brakes suddenly, bringing the car rapidly to a halt. There is negligible friction between the car seat and the box.

    1. Draw a labelled free-body diagram of **the forces acting on the child** during the time that the car is being braked.
    2. Draw a labelled free-body diagram of **the forces acting on the box** during the time that the car is being braked.
    3. What is the rate of change of the child's momentum as the car is braked to a standstill from a speed of 72 km.h^{-1} in 4 s. *Modern cars are designed with safety features (besides seat belts) to protect drivers and passengers during collisions e.g. the crumple zones on the car's body. Rather than remaining rigid during a collision, the crumple zones allow the car's body to collapse steadily.*
    4. State Newton's second law of motion.
    5. Explain how the crumple zone on a car reduces the force of impact on it during a collision.

25. [SC 2003/11 HG1]The total mass of a lift together with its load is 1 200 kg. It is moving downwards at a constant velocity of 9 m·s^{-1}.

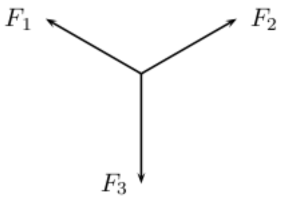
    

    1. What will be the magnitude of the force exerted by the cable on the lift while it is moving downwards at constant velocity? Give an explanation for your answer. *The lift is now uniformly brought to rest over a distance of 18 m.*
    2. Calculate the magnitude of the acceleration of the lift.
    3. Calculate the magnitude of the force exerted by the cable while the lift is being brought to rest.

26. A driving force of 800 N acts on a car of mass 600 kg.

    1. Calculate the car's acceleration.
    2. Calculate the car's speed after 20 s.
    3. Calculate the new acceleration if a frictional force of 50 N starts to act on the car after 20 s.
    4. Calculate the speed of the car after another 20 s (i.e. a total of 40 s after the start).

27. [IEB 2002/11 HG1 - A Crate on an Inclined Plane] Elephants are being moved from the Kruger National Park to the Eastern Cape. They are loaded into crates that are pulled up a ramp (an inclined plane) on frictionless rollers. The diagram shows a crate being held stationary on the ramp by means of a rope parallel to the ramp. The tension in the rope is 5 000 N.



1. Explain how one can deduce the following: “The forces acting on the crate are in equilibrium”.
2. Draw a labelled free-body diagram of the forces acting on the crane and elephant. (Regard the crate and elephant as one object, and represent them as a dot. Also show the relevant angles between the forces.)
3. The crate has a mass of 800 kg. Determine the mass of the elephant.
4. The crate is now pulled up the ramp at a constant speed. How does the crate being pulled up the ramp at a constant speed affect the forces acting on the crate and elephant? Justify your answer, mentioning any law or principle that applies to this situation.

28. [IEB 2002/11 HG1 - Car in Tow] Car A is towing Car B with a light tow rope. The cars move along a straight, horizontal road.

    1. Write down a statement of Newton's Second Law of Motion (in words).
    2. As they start off, Car A exerts a forwards force of 600 N at its end of the tow rope. The force of friction on Car B when it starts to move is 200 N. The mass of Car B is 1 200 kg. Calculate the acceleration of Car B.
    3. After a while, the cars travel at constant velocity. The force exerted on the tow rope is now 300 N while the force of friction on Car B increases. What is the magnitude and direction of the force of friction on Car B now?
    4. Towing with a rope is very dangerous. A solid bar should be used in preference to a tow rope. This is especially true should Car A suddenly apply brakes. What would be the advantage of the solid bar over the tow rope in such a situation?
    5. The mass of Car A is also 1 200 kg. Car A and Car B are now joined by a solid tow bar and the total braking force is 9 600 N. Over what distance could the cars stop from a velocity of 20 m·s^{-1}?

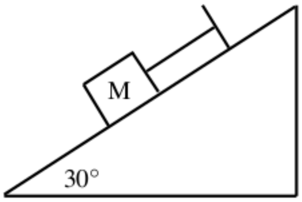
29. [IEB 2001/11 HG1] - **Testing the Brakes of a Car** A braking test is carried out on a car travelling at 20 m·s $^{-1}$. A braking distance of 30 m is measured when a braking force of 6 000 N is applied to stop the car.

    1. Calculate the acceleration of the car when a braking force of 6 000 N is applied.
    2. **Show** that the mass of this car is 900 kg.
    3. How long (in s) does it take for this car to stop from 20 m·s^{-1}under the braking action described above?
    4. A trailer of mass 600 kg is attached to the car and the braking test is repeated from 20 m·s^{-1}using the same braking force of 6 000 N. How much longer will it take to stop the car with the trailer in tow?

30. [IEB 2001/11 HG1] A rocket takes off from its launching pad, accelerating up into the air. Which of the following statements best describes the reason for the upward acceleration of the rocket?

    1. **A**The force that the atmosphere (air) exerts underneath the rocket is greater than the weight of the rocket.
    2. **B**The force that the ground exerts on the rocket is greater than the weight of the rocket.
    3. **C**The force that the rocket exerts on the escaping gases is less than the weight of the rocket.
    4. **D**The force that the escaping gases exerts on the rocket is greater than the weight of the rocket.

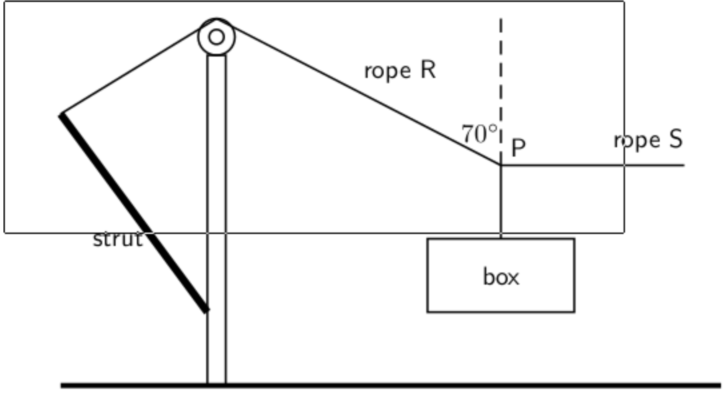
31. [IEB 2005/11 HG] A box is held stationary on a smooth plane that is inclined at angle θ to the horizontal.



F is the force exerted by a rope on the box. w is the weight of the box and N is the normal force of the plane on the box. Which of the following statements is correct?

1. **A** $\tan\theta = \frac{F}{w}$
2. **B** $\tan\theta = \frac{F}{N}$
3. **C** $\cos\theta = \frac{F}{w}$
4. **D** $\sin\theta = \frac{N}{w}$

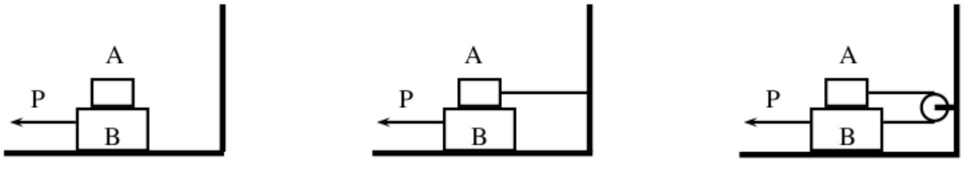
32. [SC 2001/11 HG1] As a result of three forces F_1, F_2 and F_3 acting on it, an object at point P is in equilibrium.



Which of the following statements is **not true** with reference to the three forces?

1. The resultant of forces F_1, F_2 and F_3 is zero.
2. Forces F_1, F_2 and F_3 lie in the same plane.
3. Forces F_3 is the resultant of forces F_1 and F_2.
4. The sum of the components of all the forces in any chosen direction is zero.

33. A block of mass M is held stationary by a rope of negligible mass. The block rests on a frictionless plane which is inclined at 30° to the horizontal.

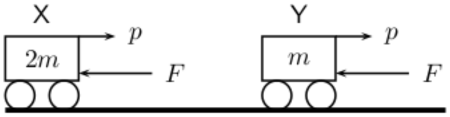


(a) (b) (c)

1. Draw a labelled force diagram which shows all the forces acting on the block.
2. Resolve the force due to gravity into components that are parallel and perpendicular to the plane.
3. Calculate the weight of the block when the force in the rope is 8N.

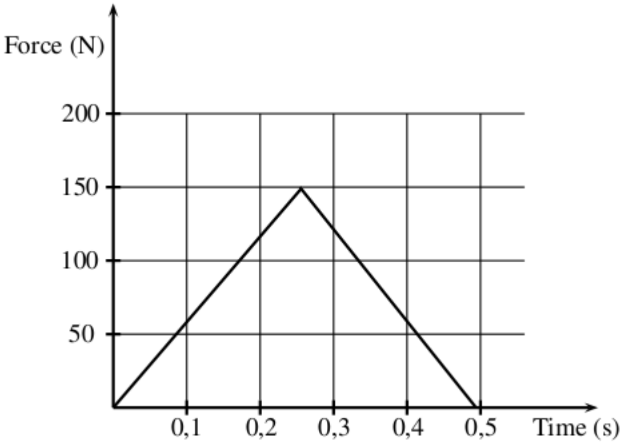
34. [SC 2003/11] A heavy box, mass m, is lifted by means of a rope R which passes over a pulley fixed to a pole. A second rope S, tied to rope R at point P, exerts a horizontal force and pulls the box to the right. After lifting

the box to a certain height, the box is held stationary as shown in the sketch below. Ignore the masses of the ropes. The tension in rope R is 5 850 N.



1. Draw a diagram (with labels) of all the forces acting at the point P, when P is in equilibrium.
2. By resolving the force exerted by rope R into components, calculate the …
    1. magnitude of the force exerted by rope S.
    2. mass, m, of the box.
3. Will the tension in rope R, increase, decrease or remain the same if rope S is pulled further to the right (the length of rope R remains the same)? Give a reason for your choice.

35. A tow truck attempts to tow a broken down car of mass 400 kg. The coefficient of static friction is 0,60 and the coefficient of kinetic (dynamic) friction is 0,4. A rope connects the tow truck to the car. Calculate the force required:
    1. to just move the car if the rope is parallel to the road.
    2. to keep the car moving at constant speed if the rope is parallel to the road.
    3. to just move the car if the rope makes an angle of 30° to the road.
    4. to keep the car moving at constant speed if the rope makes an angle of 30° to the road.
36. A crate weighing 2000 N is to be lowered at constant speed down skids 4 m long, from a truck 2 m high.
    1. If the coefficient of sliding friction between the crate and the skids is 0,30, will the crate need to be pulled down or held back?
    2. How great is the force needed parallel to the skids?
37. Block A in the figures below weighs 4 N and block B weighs 8 N. The coefficient of kinetic friction between all surfaces is 0,25. Find the force P necessary to drag block B to the left at constant speed if
    1. A rests on B and moves with it
    2. A is held at rest
    3. A and B are connected by a light flexible cord passing around a fixed frictionless pulley



Gravitation

1. [SC 2003/11]An object attracts another with a gravitational force F. If the distance between the centres of the two objects is now decreased to a third ($\frac{1}{3}$) of the original distance, the force of attraction that the one object would exert on the other would become...
    1. **A**$\frac{1}{9}F$
    2. **B**$\frac{1}{3}F$
    3. **C**$3F$
    4. **D**$9F$
2. [SC 2003/11] An object is dropped from a height of 1 km above the Earth. If air resistance is ignored, the acceleration of the object is dependent on the $\cdots$
    1. **A**mass of the object
    2. **B**radius of the earth
    3. **C**mass of the earth
    4. **D**weight of the object
3. A man has a mass of 70 kg on Earth. He is walking on a new planet that has a mass four times that of the Earth and the radius is the same as that of the Earth (M_E = 6 x 10^{24} kg, r_E = 6 x 10^6 m)
    1. Calculate the force between the man and the Earth.
    2. What is the man's mass on the new planet?
    3. Would his weight be bigger or smaller on the new planet? Explain how you arrived at your answer.
4. Calculate the distance between two objects, 5000 kg and 6 x 10^{12} kg respectively, if the magnitude of the force between them is 3 x $10^{?8}$ N.
5. Calculate the mass of the Moon given that an object weighing 80 N on the Moon has a weight of 480 N on Earth and the radius of the Moon is 1,6 x 10^{16} m.
6. The following information was obtained from a free-fall experiment to determine the value of g with a pendulum. Average falling distance between marks = 920 mm Time taken for 40 swings = 70 s Use the data to calculate the value of g.
7. An astronaut in a satellite 1600 km above the Earth experiences gravitational force of the magnitude of 700 N on Earth. The Earth's radius is 6400 km. Calculate
    1. The magnitude of the gravitational force which the astronaut experiences in the satellite.
    2. The magnitude of the gravitational force on an object in the satellite which weighs 300 N on Earth.
8. An astronaut of mass 70 kg on Earth lands on a planet which has half the Earth's radius and twice its mass. Calculate the magnitude of the force of gravity which is exerted on him on the planet.
9. Calculate the magnitude of the gravitational force of attraction between two spheres of lead with a mass of 10 kg and 6 kg respectively if they are placed 50 mm apart.
10. The gravitational force between two objects is 1200 N. What is the gravitational force between the objects if the mass of each is doubled and the distance between them halved?
11. Calculate the gravitational force between the Sun with a mass of 2 x 10^{30} kg and the Earth with a mass of 6 x 10^{24} kg if the distance between them is 1,4 x 10^8 km.
12. How does the gravitational force of attraction between two objects change when
    1. the mass of each object is doubled.
    2. the distance between the centres of the objects is doubled.
    3. the mass of one object is halved, and the distance between the centres of the objects is halved.
13. Read each of the following statements and say whether you agree or not. Give reasons for your answer and rewrite the statement if necessary:
    1. The gravitational acceleration g is constant.
    2. The weight of an object is independent of its mass.
    3. G is dependent on the mass of the object that is being accelerated.

14. An astronaut weighs 750 N on the surface of the Earth.
    1. What will his weight be on the surface of Saturn, which has a mass 100 times greater than the Earth, and a radius 5 times greater than the Earth?
    2. What is his mass on Saturn?
15. A piece of space garbage is at rest at a height 3 times the Earth's radius above the Earth's surface. Determine its acceleration due to gravity. Assume the Earth's mass is 6,0 x 10^{24} kg and the Earth's radius is 6400 km.
16. Your mass is 60 kg in Paris at ground level. How much less would you weigh after taking a lift to the top of the Eiffel Tower, which is 405 m high? Assume the Earth's mass is 6,0 x 10^{24} kg and the Earth's radius is 6400 km.
17.
    1. State Newton's Law of Universal Gravitation.
    2. Use Newton's Law of Universal Gravitation to determine the magnitude of the acceleration due to gravity on the Moon. The mass of the Moon is $7{,}40 \times 10^{22}$ kg. The radius of the Moon is $1{,}74 \times 10^{6}$ m.
    3. Will an astronaut, kitted out in his space suit, jump higher on the Moon or on the Earth? Give a reason for your answer.

Momentum

1. [SC 2003/11]A projectile is fired vertically upwards from the ground. At the highest point of its motion, the projectile explodes and separates into two pieces of equal mass. If one of the pieces is projected vertically upwards after the explosion, the second piece will ...
    1. **A**drop to the ground at zero initial speed.
    2. **B**be projected downwards at the same initial speed at the first piece.
    3. **C**be projected upwards at the same initial speed as the first piece.
    4. **D**be projected downwards at twice the initial speed as the first piece.
2. [IEB 2004/11 HG1] A ball hits a wall horizontally with a speed of 15 $\text{m}\cdot\text{s}^{-1}$. It rebounds horizontally with a speed of 8 $\text{m}\cdot\text{s}^{-1}$. Which of the following statements about the system of the ball and the wall is **true**?
    1. **A**The total linear momentum of the system is not conserved during this collision.
    2. **B**The law of conservation of energy does not apply to this system.
    3. **C**The change in momentum of the wall is equal to the change in momentum of the ball.
    4. **D**Energy is transferred from the ball to the wall.
3. [IEB 2001/11 HG1] A block of mass M collides with a stationary block of mass 2M. The two blocks move off together with a velocity of v. What is the velocity of the block of mass M immediately **before** it collides with the block of mass 2M?
    1. **A**v
    2. **B**2v
    3. **C**3v
    4. **D**4v
4. [IEB 2003/11 HG1] A cricket ball and a tennis ball move horizontally towards you with the same momentum. A cricket ball has greater mass than a tennis ball. You apply the same force in stopping each ball. How does the time taken to stop each ball compare?
    1. **A**It will take longer to stop the cricket ball.
    2. **B**It will take longer to stop the tennis ball.
    3. **C**It will take the same time to stop each of the balls.
    4. **D**One cannot say how long without knowing the kind of collision the ball has when stopping.

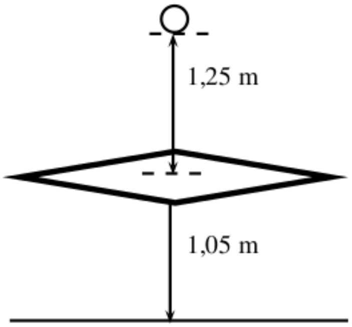
5. [IEB 2004/11 HG1] Two identical billiard balls collide head-on with each other. The first ball hits the second ball with a speed of V, and the second ball hits the first ball with a speed of 2V. After the collision, the first ball moves off in the opposite direction with a speed of 2V. Which expression correctly gives the speed of the second ball after the collision?

    1. **A**V
    2. **B**2V
    3. **C**3V
    4. **D**4V

6. [SC 2002/11 HG1] Which one of the following physical quantities is the same as the rate of change of momentum?

    1. **A**resultant force
    2. **B**work
    3. **C**power
    4. **D**impulse

7. [IEB 2005/11 HG] Cart X moves along a smooth track with momentum p. A resultant force F applied to the cart stops it in time t. Another cart Y has only half the mass of X, but it has the same momentum p.

    In what time will cart Y be brought to rest when the same resultant force F acts on it?

    1. **A**$\frac{1}{2}t$
    2. **B**t
    3. **C**$2t$
    4. **D**$4t$

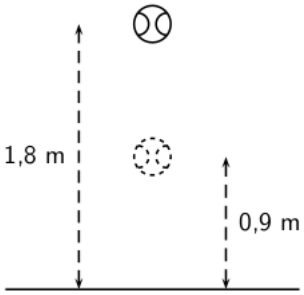
8. [SC 2002/03 HG1] A ball with mass m strikes a wall perpendicularly with a speed, v. If it rebounds in the opposite direction with the same speed, v, the magnitude of the change in momentum will be ...

    1. **A**$2mv$
    2. **B**mv
    3. **C**$\frac{1}{2}mv$
    4. **D**$0\ mv$

9. Show that impulse and momentum have the same units.
10. A golf club exerts an average force of 3 kN on a ball of mass 0,06 kg. If the golf club is in contact with the golf ball for 5 x 10^{-4} seconds, calculate

    1. the change in the momentum of the golf ball.
    2. the velocity of the golf ball as it leaves the club.

11. During a game of hockey, a player strikes a stationary ball of mass 150 g. The graph below shows how the force of the ball varies with the time.



   1. What does the area under this graph represent?
   2. Calculate the speed at which the ball leaves the hockey stick.
   3. The same player hits a practice ball of the same mass, but which is made from a softer material. The hit is such that the ball moves off with the same speed as before. How will the **area**, the **height** and the **base** of the triangle that forms the graph, compare with that of the original ball?

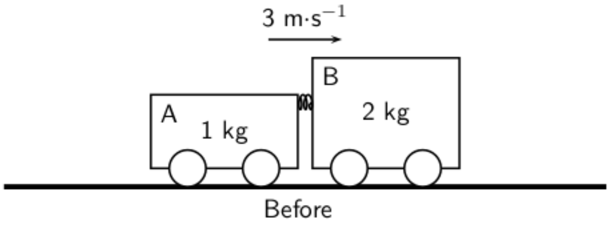
12. The fronts of modern cars are deliberately designed in such a way that in case of a head-on collision, the front would crumple. Why is it desirable that the front of the car should crumple?
13. A ball of mass 100 g strikes a wall horizontally at 10 m·s^{-1}and rebounds at 8 m·s^{-1}. It is in contact with the wall for 0,01 s.

    1. Calculate the average force exerted by the wall on the ball.
    2. Consider a lump of putty also of mass 100 g which strikes the wall at 10 m·s^{-1}and comes to rest in 0,01 s against the surface. Explain qualitatively (no numbers) whether the force exerted on the putty will be less than, greater than of the same as the force exerted on the ball by the wall. Do not do any calculations.

14. Shaun swings his cricket bat and hits a stationary cricket ball vertically upwards so that it rises to a height of 11,25 m above the ground. The ball has a mass of 125 g. Determine

    1. the speed with which the ball left the bat.
    2. the impulse exerted by the bat on the ball.
    3. the impulse exerted by the ball on the bat.
    4. for how long the ball is in the air.

15. A glass plate is mounted horizontally 1,05 m above the ground. An iron ball of mass 0,4 kg is released from rest and falls a distance of 1,25 m before striking the glass plate and breaking it. The total time taken from release to hitting the ground is recorded as 0,80 s. Assume that the time taken to break the plate is negligible.

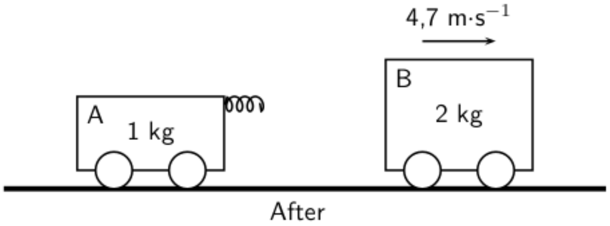


    1. Calculate the speed at which the ball strikes the glass plate.
    2. Show that the speed of the ball immediately after breaking the plate is 2,0 m·s^{-1}.
    3. Calculate the magnitude and give the direction of the change of momentum which the ball experiences during its contact with the glass plate.
    4. Give the magnitude and direction of the impulse which the glass plate experiences when the ball hits it.

16. [SC 2004/11 HG1]A cricket ball, mass 175 g is thrown directly towards a player at a velocity of 12 m·s^{-1}. It is hit back in the opposite direction with a velocity of 30 m·s^{-1}. The ball is in contact with the bat for a period of 0,05 s.

1. Calculate the impulse of the ball.
2. Calculate the magnitude of the force exerted by the bat on the ball.

17. [IEB 2005/11 HG1] A ball bounces to a vertical height of 0,9 m when it is dropped from a height of 1,8 m. It rebounds immediately after it strikes the ground, and the effects of air resistance are negligible.

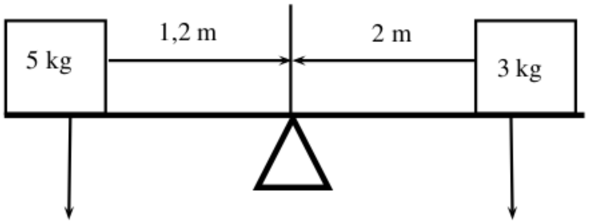


   1. How long (in s) does it take for the ball to hit the ground after it has been dropped?
   2. At what speed does the ball strike the ground?
   3. At what speed does the ball rebound from the ground?
   4. How long (in s) does the ball take to reach its maximum height after the bounce?
   5. Draw a velocity-time graph for the motion of the ball from the time it is dropped to the time when it rebounds to 0,9 m. Clearly, show the following on the graph:
      1. the time when the ball hits the ground
      2. the time when it reaches 0,9 m
      3. the velocity of the ball when it hits the ground, and
      4. the velocity of the ball when it rebounds from the ground.

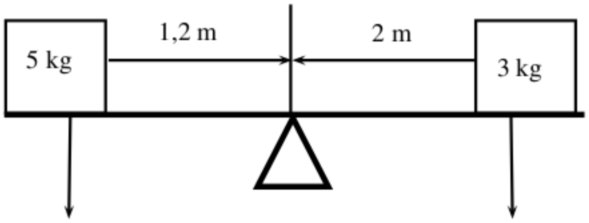
18. [SC 2002/11 HG1] In a railway shunting yard, a locomotive of mass 4 000 kg, travelling due east at a velocity of 1,5 m·s^{-1}, collides with a stationary goods wagon of mass 3 000 kg in an attempt to couple with it. The coupling fails and instead the goods wagon moves due east with a velocity of 2,8 m·s^{-1}.

   1. Calculate the magnitude and direction of the velocity of the locomotive immediately after collision.
   2. Name and state in words the law you used to answer question [link]

19. [SC 2005/11 SG1] A combination of trolley A (fitted with a spring) of mass 1 kg, and trolley B of mass 2 kg, moves to the right at 3 m·s^{-1} along a frictionless, horizontal surface. The spring is kept compressed between the two trolleys.



While the combination of the two trolleys is moving at 3 m·s^{-1} , the spring is released and when it has expanded completely, the 2 kg trolley is then moving to the right at 4,7 m·s^{-1} as shown below.



   1. State, in words, the principle of *conservation of linear momentum*.

    2. Calculate the magnitude and direction of the velocity of the 1 kg trolley immediately after the spring has expanded completely.

20. [IEB 2002/11 HG1] A ball bounces back from the ground. Which of the following statements is true of this event?

    1. The magnitude of the change in momentum of the ball is equal to the magnitude of the change in momentum of the Earth.
    2. The magnitude of the impulse experienced by the ball is greater than the magnitude of the impulse experienced by the Earth.
    3. The speed of the ball before the collision will always be equal to the speed of the ball after the collision.
    4. Only the ball experiences a change in momentum during this event.

21. [SC 2002/11 SG] A boy is standing in a small stationary boat. He throws his schoolbag, mass 2 kg, horizontally towards the jetty with a velocity of 5 m·s^{-1}. The *combined mass* of the boy and the boat is 50 kg.

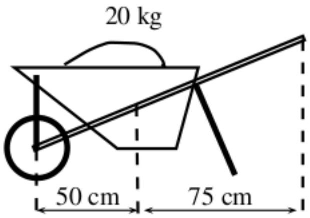
    1. Calculate the magnitude of the horizontal momentum of the bag immediately after the boy has thrown it.
    2. Calculate the velocity (magnitude and direction) of the *boat-and-boy* immediately after the bag is thrown.

Torque and levers

1. State whether each of the following statements are true or false. If the statement is false, rewrite the statement correcting it.

    1. The torque tells us what the turning effect of a force is.
    2. To increase the mechanical advantage of a spanner you need to move the effort closer to the load.
    3. A class 2 lever has the effort between the fulcrum and the load.
    4. An object will be in equilibrium if the clockwise moment and the anticlockwise moments are equal.
    5. Mechanical advantage is a measure of the difference between the load and the effort.
    6. The force times the perpendicular distance is called the mechanical advantage.

2. Study the diagram below and determine whether the seesaw is balanced. Show all your calculations.



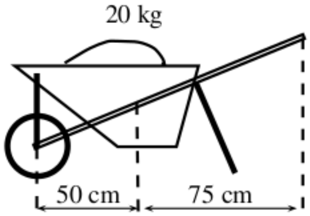
3. Two children are playing on a seesaw. Tumi has a weight of 200 N and Thandi weighs 240 N. Tumi is sitting at a distance of 1,2 m from the pivot.

    1. What type of lever is a seesaw?
    2. Calculate the moment of the force that Tumi exerts on the seesaw.
    3. At what distance from the pivot should Thandi sit to balance the seesaw?

4. An applied force of 25 N is needed to open the cap of a glass bottle using a bottle opener. The distance between the applied force and the fulcrum is 10 cm and the distance between the load and the fulcrum is 1 cm.

    1. What type of lever is a bottle opener? Give a reason for your answer.
    2. Calculate the mechanical advantage of the bottle opener.
    3. Calculate the force that the bottle cap is exerting.

5. Determine the force needed to lift the 20 kg load in the wheelbarrow in the diagram below.



6. A body builder picks up a weight of 50 N using his right hand. The distance between the body builder's hand and his elbow is 45 cm. The distance between his elbow and where his muscles are attached to his forearm is 5 cm.

   1. What type of lever is the human arm? Explain your answer using a diagram.
   2. Determine the force his muscles need to apply to hold the weight steady.

Gravity and Mechanical Energy
Veritasium video on falling objects - 1
[missing_resource: http://www.youtube.com/v/_mCC-68LyZM&rel=0]

Weight

Weight is the gravitational force that the Earth exerts on any object. The weight of an objects gives you an indication of how strongly the Earth attracts that body towards its centre. Weight is calculated as follows:

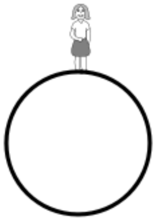
$\text{Weight} = mg$

where m = mass of the object (in kg)

and g = the acceleration due to gravity (9,8 $m{\cdot}\text{s}^{-2}$)

For example, what is Sarah's weight if her mass is $50\ \text{kg}$. Sarah's weight is calculated according to:
Equation:

$$\begin{aligned}\text{Weight} &= mg\\ &= (50\ \text{kg})\left(9,8\,\text{m}\cdot\text{s}^{-2}\right)\\ &= 490\ \text{kg}\cdot\ \text{m}\cdot\text{s}^{-2}\\ &= 490\ \text{N}\end{aligned}$$



Note: Weight is sometimes abbreviated as F_g which refers to the force of gravity. Do not use the abbreviation W for weight as it refers to `Work'.

Now, we have said that the value of g is approximately 9,8 $m\cdot s^{-2}$ on the surface of the Earth. The actual value varies slightly over the surface of the Earth. Each planet in our Solar System has its own value for g. These values are listed as multiples of g on Earth in [link]

Planet	Gravitational Acceleration (multiples of g on Earth)
Mercury	0,376
Venus	0,903

Earth	1
Mars	0,38
Jupiter	2,34
Saturn	1,16
Uranus	1,15
Neptune	1,19
Pluto	0,066

A list of the gravitational accelerations at the surfaces of each of the planets in our solar system. Values are listed as multiples of *g* on Earth. **Note**: The "surface" is taken to mean the cloud tops of the gas giants (Jupiter, Saturn, Uranus and Neptune).

Exercise:
Determining mass and weight on other planets

Problem: Sarah's mass on Earth is $50\ \text{kg}$. What is her mass and weight on Mars?

Solution:

Determine what information is given and what is asked

m (on Earth) = $50\ \text{kg}$

m (on Mars) = ?

Weight (on Mars) = ?

Calculate her mass on Mars

Sarah's mass does not change because she is still made up of the same amount of matter. Her mass on Mars is therefore $50\ \text{kg}$.

Calculate her weight on Mars

Equation:

$$\begin{aligned}\text{Sarah's weight} &= 50 \times 0,38 \times 9,8 \\ &= 186,2\ \text{N}\end{aligned}$$

Differences between Mass and Weight

Mass is measured in kilograms (kg) and is the amount of matter in an object. An object's mass does not change unless matter is added or removed from the object.

The differences between mass and weight can be summarised in the following table:

Mass	Weight
1. is a measure of how much matter there is in an object.	1. is the force with which the Earth attracts an object.

2. is measured in kilograms.	2. is measured in newtons
3. is the same on any planet.	3. is different on different planets.
4. is a scalar.	4. is a vector.

Weight

1. A bag of sugar has a mass of 1 kg. How much does it weigh:

   a. on Earth?
   b. on Jupiter?
   c. on Pluto?

   Click here for the solution
2. Neil Armstrong was the first man to walk on the surface of the Moon. The gravitational acceleration on the Moon is $\frac{1}{6}$ of the gravitational acceleration on Earth, and there is negligible gravitational acceleration in outer space. If Neil's mass was 90 kg, what was his weight:

   a. on Earth?
   b. on the Moon?
   c. in outer space?

   Click here for the solution
3. A monkey has a mass of 15 kg on Earth. The monkey travels to Mars. What is his mass and weight on Mars?
   Click here for the solution
4. Determine your mass by using a bathroom scale and calculate your weight for each planet in the Solar System, using the values given in [link]
   Click here for the solution

Acceleration due to Gravity

Gravitational Fields

A *field* is a region of space in which a mass experiences a force. Therefore, a *gravitational* field is a region of space in which a mass experiences a gravitational force.

Free fall

Note: Free fall is motion in the Earth's gravitational field when no other forces act on the object.

Free fall is the term used to describe a special kind of motion in the Earth's gravitational field. Free fall is motion in the Earth's gravitational field when no other forces act on the object. It is basically an ideal situation, since in reality, there is always some air friction which slows down the motion.

Experiment : Acceleration due to Gravity

Aim: Investigating the acceleration of two different objects during free fall.

Apparatus: Tennis ball and a sheet of A4 paper.

Method:

1. Hold the tennis ball and sheet of paper (horizontally) the same distance from the ground. Which one would strike the ground first if both were dropped?

2. Drop both objects and observe. Explain your observations.
3. Now crumple the paper into a ball, more or less the same size as the tennis ball. Drop the paper and tennis ball again and observe. Explain your observations.
4. Why do you think the two situations are different?
5. Compare the value for the acceleration due to gravity of the tennis ball to the crumpled piece of paper.
6. Predict what will happen if an iron ball and a tennis ball of the same size are dropped from the same height. What will the values for their acceleration due to gravity be?

If a metal ball and tennis ball (of the same size) were dropped from the same height, both would reach the ground at the same time. It does not matter that the one ball is heavier than the other. The acceleration of an object due to gravity is independent of the mass of the object. It does not matter what the mass of the object is.

The shape of the object, however, is important. The sheet of paper took much longer to reach the ground than the tennis ball. This is because the effect of air friction on the paper was much greater than the air friction on the tennis ball.

If we lived in a world where there was no air resistance, the A4 sheet of paper and the tennis ball would reach the ground at the same time. This happens in outer space or in a vaccuum.

Galileo Galilei, an Italian scientist, studied the motion of objects. The following case study will tell you more about one of his investigations.

Case Study : Galileo Galilei

In the late sixteenth century, it was generally believed that heavier objects would fall faster than lighter objects. The Italian scientist Galileo Galilei thought differently. Galileo hypothesized that two objects would fall at the same rate regardless of their mass. Legend has it that in 1590, Galileo planned out an experiment. He climbed to the top of the Leaning Tower of Pisa and dropped several large objects to test his theory. He wanted to show that two different objects fall at the same rate (as long as we ignore air resistance). Galileo's experiment proved his hypothesis correct; the acceleration of a falling object is independent of the object's mass.

A few decades after Galileo, Sir Isaac Newton would show that acceleration depends upon both force and mass. While there is greater force acting on a larger object, this force is canceled out by the object's greater mass. Thus two objects will fall (actually they are pulled) to the earth at exactly the same rate.

Questions: Read the case study above and answer the following questions.

1. Divide into pairs and explain Galileo's experiment to your friend.
2. Write down an aim and a hypothesis for Galileo's experiment.
3. Write down the result and conclusion for Galileo's experiment.

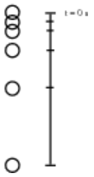
Research Project : Experimental Design

Design an experiment similar to the one done by Galileo to prove that the acceleration due to gravity of an object is independent of the object's mass. The investigation must be such that you can perform it at home or at school. Bring your apparatus to school and perform the experiment. Write it up and hand it in for assessment.

Case Study : Determining the acceleration due to gravity 1

Study the set of photographs alongside showing the position of a ball being dropped from a height at constant time intervals. The distance of the ball from the starting point in each consecutive image is observed to be: $x_1 = 0$ cm, $x_2 = 4,9$ cm, $x_3 = 19,6$ cm, $x_4 = 44,1$ cm, $x_5 = 78,4$ cm and $x_6 = 122,5$ cm. Answer the following questions:

1. Determine the time between each picture if the frequency of the exposures were 10 Hz.
2. Calculate the velocity, v_2, of the ball between positions 1 and 3.
   Equation:
   $$v_2 = \frac{x_3 - x_1}{t_3 - t_1}$$
3. Calculate the velocity, v_5, of the ball between positions 4 and 6.
   Equation:
   $$v_5 = \frac{x_6 - x_4}{t_6 - t_4}$$
4. Calculate the acceleration the ball between positions 2 and 5.
   Equation:
   $$a = \frac{v_5 - v_2}{t_5 - t_2}$$
5. Compare your answer to the value for the acceleration due to gravity (9,8 m·s^{-2}).



The acceleration due to gravity is constant. This means we can use the equations of motion under constant acceleration that we derived in motion in one dimension to describe the motion of an object in free fall. The equations are repeated here for ease of use.

Equation:

$$\begin{aligned}
v_i &= \text{initial velocity}\left(\text{m}\cdot\text{s}^{-1}\right)\text{ at t} = 0\text{s} \\
v_f &= \text{final velocity}\left(\text{m}\cdot\text{s}^{-1}\right)\text{at time t} \\
\mathit{\Delta} x &= \text{displacement}(\text{m}) \\
t &= \text{time}(\text{s}) \\
\mathit{\Delta} t &= \text{time interval}(\text{s}) \\
g &= \text{acceleration}\left(\text{m}\cdot\text{s}^{-2}\right)
\end{aligned}$$

Equation:

$$v_f = v_i + gt$$

Equation:

$$\Delta x = \frac{(v_i + v_f)}{2} t$$

Equation:

$$\Delta x = v_i t + \frac{1}{2} g t^2$$

Equation:

$$v_f^2 = v_i^2 + 2g\Delta x$$

Experiment : Determining the acceleration due to gravity 2

Work in groups of at least two people.

Aim: To determine the acceleration of an object in freefall.

Apparatus: Large marble, two stopwatches, measuring tape.

Method:

1. Measure the height of a door, from the top of the door to the floor, exactly. Write down the measurement.
2. One person must hold the marble at the top of the door. Drop the marble to the floor at the same time as he/she starts the first stopwatch.
3. The second person watches the floor and starts his stopwatch when the marble hits the floor.
4. The two stopwatches are stopped together and the two times substracted. The difference in time will give the time taken for the marble to fall from the top of the door to the floor.
5. Design a table to show the results of your experiment. Choose appropriate headings and units.
6. Choose an appropriate equation of motion to calculate the acceleration of the marble. Remember that the marble starts from rest and that it's displacement was determined in the first step.
7. Write a conclusion for your investigation.
8. Answer the following questions:
   a. Why do you think two stopwatches were used in this investigation?
   b. Compare the value for acceleration obtained in your investigation with the value of acceleration due to gravity ($9{,}8\ m \cdot s^{-2}$). Explain your answer.

Exercise:
A freely falling ball

Problem:

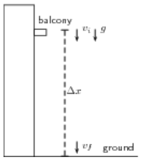
A ball is dropped from the balcony of a tall building. The balcony is $15\ m$ above the ground. Assuming gravitational acceleration is $9{,}8\ m \cdot s^{-2}$, find:

1. the time required for the ball to hit the ground, and
2. the velocity with which it hits the ground.

Solution:

Draw a rough sketch of the problem

It always helps to understand the problem if we draw a picture like the one below:



Identify what information is given and what is asked for

Equation:

We have these quantities:

$$\begin{aligned} \Delta x &= 15\ \text{m} \\ v_i &= 0\ \text{m}\cdot\text{s}^{-1} \\ g &= 9{,}8\ \text{m}\cdot\text{s}^{-2} \end{aligned}$$

Choose up or down as the positive direction.

Since the ball is falling, we choose down as positive. This means that the values for v_i, Δx and a will be positive.

Choose the most appropriate equation.

We can use [link] to find the time: $\Delta x = v_i t + \frac{1}{2} g t^2$

Use the equation to find .

Equation:

$$\begin{aligned} \Delta x &= v_i t + \frac{1}{2} g t^2 \\ 15 &= (0)t + \frac{1}{2}(9,8)(t)^2 \\ 15 &= 4,9\ t^2 \\ t^2 &= 3,0612... \\ t &= 1,7496... \\ t &= 1,75\ s \end{aligned}$$

Find the final velocity .

Equation:

Using [link] to find v_f:

$$\begin{aligned} v_f &= v_i + g t \\ v_f &= 0 + (9,8)(1,7496...) \\ v_f &= 17,1464... \end{aligned}$$

Remember to add the direction: $v_f = 17{,}15\, m \cdot s^{-1}$ downwards.

By now you should have seen that free fall motion is just a special case of motion with constant acceleration, and we use the same equations as before. The only difference is that the value for the acceleration, a, is always equal to the value of gravitational acceleration, g. In the equations of motion we can replace a with g.

Gravitational Acceleration

1. A brick falls from the top of a 5 m high building. Calculate the velocity with which the brick reaches the ground. How long does it take the brick to reach the ground?
   Click here for the solution
2. A stone is dropped from a window. It takes the stone 1,5 s to reach the ground. How high above the ground is the window?
   Click here for the solution
3. An apple falls from a tree from a height of 1,8 m. What is the velocity of the apple when it reaches the ground?
   Click here for the solution

Potential Energy

The potential energy of an object is generally defined as the energy an object has because of its position relative to other objects that it interacts with. There are different kinds of potential energy such as gravitional potential energy, chemical potential energy, electrical potential energy, to name a few. In this section we will be looking at gravitational potential energy.

Potential energy

> Potential energy is the energy an object has due to its position or state.

Gravitational potential energy is the energy of an object due to its position above the surface of the Earth. The symbol PE is used to refer to gravitational potential energy. You will often find that the words potential energy are used where *gravitational* potential energy is meant. We can define potential energy (or gravitational potential energy, if you like) as:

Equation:

$$PE = mgh$$

where PE = potential energy measured in joules (J)

m = mass of the object (measured in kg)

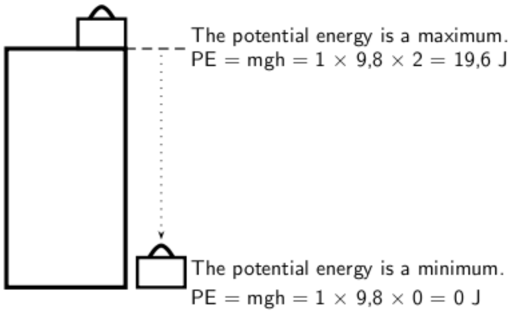
g = gravitational acceleration ($9{,}8\ m \cdot s^{-2}$)

h = perpendicular height from the reference point (measured in m)

A suitcase, with a mass of 1 kg, is placed at the top of a $2\ m$ high cupboard. By lifting the suitcase against the force of gravity, we give the suitcase potential energy. This potential energy can be calculated using [link].

If the suitcase falls off the cupboard, it will lose its potential energy. Halfway down the cupboard, the suitcase will have lost half its potential energy and will have only $9{,}8\ J$ left. At the bottom of the cupboard the suitcase will have lost all it's potential energy and it's potential energy will be equal to zero.

Objects have maximum potential energy at a maximum height and will lose their potential energy as they fall.


The potential energy is a maximum.
PE = mgh = 1 × 9,8 × 2 = 19,6 J
The potential energy is a minimum.
PE = mgh = 1 × 9,8 × 0 = 0 J

Exercise:
Gravitational potential energy

Problem:

A brick with a mass of 1 kg is lifted to the top of a $4\ m$ high roof. It slips off the roof and falls to the ground. Calculate the potential energy of the brick at the top of the roof and on the ground once it has fallen.

Solution:

Analyse the question to determine what

information is provided	◦ The mass of the brick is $m = 1\ \text{kg}$ ◦ The height lifted is $h = 4\,m$	All quantities are in SI units.
Analyse the question to determine what is being asked	◦ We are asked to find the gain in potential energy of the brick as it is lifted onto the roof. ◦ We also need to calculate the potential energy once the brick is on the ground again.	
Identify the type of potential energy involved	Since the block is being lifted we are dealing with gravitational potential energy. To work out PE, we need to know the mass of the object and the height lifted. As both of these are given, we just substitute them into the equation for PE.	
Substitute and calculate	**Equation:** $$\begin{aligned} PE &= mgh \\ &= (1)(9,8)(4) \\ &= 39,2\,\text{J} \end{aligned}$$	

Gravitational Potential Energy

1. Describe the relationship between an object's gravitational potential energy and its:
   a. mass and
   b. height above a reference point.

   Click here for the solution
2. A boy, of mass $30\ \text{kg}$, climbs onto the roof of a garage. The roof is $2{,}5\,m$ from the ground. He now jumps off the roof and lands on the ground.
   a. How much potential energy has the boy gained by climbing on the roof?
   b. The boy now jumps down. What is the potential energy of the boy when he is $1\,m$ from the ground?
   c. What is the potential energy of the boy when he lands on the ground?

   Click here for the solution
3. A hiker walks up a mountain, $800\,m$ above sea level, to spend the night at the top in the first overnight hut. The second day he walks to the second overnight hut, $500\,m$ above sea level. The third day he returns to his starting point, $200\,m$ above sea level.
   a. What is the potential energy of the hiker at the first hut (relative to sea level)?
   b. How much potential energy has the hiker lost during the second day?
   c. How much potential energy did the hiker have when he started his journey (relative to sea level)?
   d. How much potential energy did the hiker have at the end of his journey?

   Click here for the solution

Kinetic Energy

Kinetic Energy

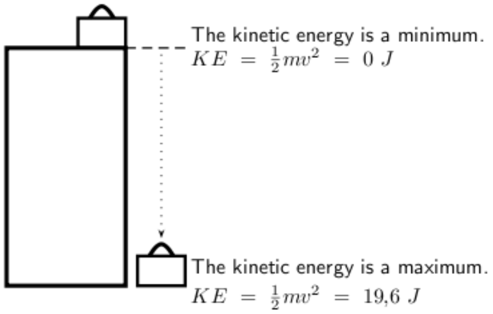
Kinetic energy is the energy an object has due to its motion.

Kinetic energy is the energy an object has because of its motion. This means that any moving object has kinetic energy. The faster it moves, the more kinetic energy it has. Kinetic energy (KE) is therefore dependent on the velocity of the object. The mass of the object also plays a role. A truck of $2\ 000\ \text{kg}$, moving at $100\,\text{km}\cdot\text{hr}^{-1}$, will have more kinetic energy than a car of $500\ \text{kg}$, also moving at $100\,\text{km}\cdot\text{hr}^{-1}$. Kinetic energy is defined as:

Equation:

$$KE = \frac{1}{2}mv^2$$

Consider the $1\ \text{kg}$ suitcase on the cupboard that was discussed earlier. When the suitcase falls, it will gain velocity (fall faster), until it reaches the ground with a maximum velocity. The suitcase will not have any kinetic energy when it is on top of the cupboard because it is not moving. Once it starts to fall it will gain kinetic energy, because it gains velocity. Its kinetic energy will increase until it is a maximum when the suitcase reaches the ground.


The kinetic energy is a minimum.
$KE = \frac{1}{2}mv^2 = 0\ J$
The kinetic energy is a maximum.
$KE = \frac{1}{2}mv^2 = 19{,}6\ J$

Exercise:

Calculation of Kinetic Energy

Problem:

A $1\ \text{kg}$ brick falls off a $4\,m$ high roof. It reaches the ground with a velocity of $8{,}85\,m\cdot s^{-1}$. What is the kinetic energy of the brick when it starts to fall and when it reaches the ground?

Solution:

Analyse the question to determine what information is provided

- The mass of the rock $m = 1\ \text{kg}$
- The velocity of the rock at the bottom $v_{\text{bottom}} = 8{,}85\,m{\cdot}\text{s}^{-1}$

These are both in the correct units so we do not have to worry about unit conversions.

Analyse the question to determine what is being asked

We are asked to find the kinetic energy of the brick at the top and the bottom. From the definition we know that to work out KE, we need to know the mass and the velocity of the object and we are given both of these values.

Calculate the kinetic energy at the top

Since the brick is not moving at the top, its kinetic energy is zero.

Substitute and calculate the kinetic energy

Equation:

$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(1\ \ \text{kg})\left(8,85\ \ \text{m}\cdot\text{s}^{-1}\right)^2 \\ &= 39,2\,\text{J} \end{aligned}$$

Checking units

According to the equation for kinetic energy, the unit should be $\text{kg}\cdot m^2\cdot s^{-2}$. We can prove that this unit is equal to the joule, the unit for energy.

Equation:

$$\begin{aligned}(\text{kg})\left(\text{m}\cdot\text{s}^{-1}\right)^2 &= \left(\text{kg}\cdot\text{m}\cdot\text{s}^{-2}\right)\cdot\text{m}\\ &= \text{N}\cdot\ \text{m}\quad \left(\text{because Force(N)} = \text{mass(kg)}\times\text{acceleration}\left(\text{m}\cdot\text{s}^{-2}\right)\right)\\ &= \text{J}\qquad\quad (\text{Work(J)} = \text{Force(N)}\times\text{distance(m)})\end{aligned}$$

We can do the same to prove that the unit for potential energy is equal to the joule:

Equation:

$$\begin{aligned}(\text{kg})\left(\text{m}\cdot\text{s}^{-2}\right)(\text{m}) &= \text{N}\cdot\ \text{m}\\ &= \text{J}\end{aligned}$$

Exercise:
Mixing Units & Energy Calculations

Problem:

A bullet, having a mass of $150\,g$, is shot with a muzzle velocity of $960\,m\cdot s^{-1}$. Calculate its kinetic energy.

Solution:

Analyse the question to determine what information is provided

- We are given the mass of the bullet $m = 150\,g$. This is not the unit we want mass to be in. We need to convert to kg.
  Equation:

$$\begin{aligned}\text{Mass in grams}\div 1000 &= \text{Mass in kg}\\ 150\ g\div 1000 &= 0,150\ \ \text{kg}\end{aligned}$$

- We are given the initial velocity with which the bullet leaves the barrel, called the muzzle velocity, and it is $v = 960\,m\cdot s^{-1}$.

Analyse the question to determine what is being asked

- We are asked to find the kinetic energy.

Substitute and calculate

We just substitute the mass and velocity (which are known) into the equation for kinetic energy:

Equation:

$$\begin{aligned}KE &= \frac{1}{2}mv^2\\ &= \frac{1}{2}(0,150)(960)^2\\ &= 69\,120\,\text{J}\end{aligned}$$

Kinetic Energy

1. Describe the relationship between an object's kinetic energy and its:
   a. mass and
   b. velocity

   Click here for the solution
2. A stone with a mass of $100\ g$ is thrown up into the air. It has an initial velocity of $3\ m \cdot s^{-1}$. Calculate its kinetic energy
   a. as it leaves the thrower's hand.
   b. when it reaches its turning point.

   Click here for the solution
3. A car with a mass of $700\ \text{kg}$ is travelling at a constant velocity of $100\ \text{km} \cdot \text{hr}^{-1}$. Calculate the kinetic energy of the car.
   Click here for the solution

Mechanical Energy

Note: Mechanical energy is the sum of the gravitational potential energy and the kinetic energy.

Mechanical energy, U, is simply the sum of gravitational potential energy (PE) and the kinetic energy (KE). Mechanical energy is defined as:

Equation:

$$U = PE + KE$$

Equation:

$$\begin{aligned} U &= PE + KE \\ U &= mgh + \frac{1}{2}mv^2 \end{aligned}$$

Conservation of Mechanical Energy

The Law of Conservation of Energy states:

"Energy cannot be created or destroyed, but is merely changed from one form into another."

Conservation of Energy

> The Law of Conservation of Energy: Energy cannot be created or destroyed, but is merely changed from one form into another.

So far we have looked at two types of energy: gravitational potential energy and kinetic energy. The sum of the gravitational potential energy and kinetic energy is called the mechanical energy. In a closed system, one where there are no external forces acting, the mechanical energy will remain constant. In other words, it will not change (become more or less). This is called the Law of Conservation of Mechanical Energy and it states:

"The total amount of mechanical energy in a closed system remains constant."

Conservation of Mechanical Energy

> Law of Conservation of Mechanical Energy: The total amount of mechanical energy in a closed system remains constant.

This means that potential energy can become kinetic energy, or vise versa, but energy cannot 'dissappear'. The mechanical energy of an object moving in the Earth's gravitational field (or accelerating as a result of gravity) is constant or conserved, unless external forces, like air resistance, acts on the object.

We can now use the conservation of mechanical energy to calculate the velocity of a body in freefall and show that the velocity is independent of mass.

Show by using the law of conservation of energy that the velocity of a body in free fall is independent of its mass.

Note: In problems involving the use of conservation of energy, the path taken by the object can be ignored. The only important quantities are the object's velocity (which gives its kinetic energy) and height above the reference point (which gives its gravitational potential energy).

Note: In the absence of friction, mechanical energy is conserved and
Equation:

$$U_{\text{before}} = U_{\text{after}}$$

In the presence of friction, mechanical energy is **not** conserved. The mechanical energy lost is equal to the work done against friction.
Equation:

$$\Delta U = U_{\text{before}} - U_{\text{after}} = \text{work done against friction}$$

In general, mechanical energy is conserved in the absence of external forces. Examples of external forces are: applied forces, frictional forces and air resistance.

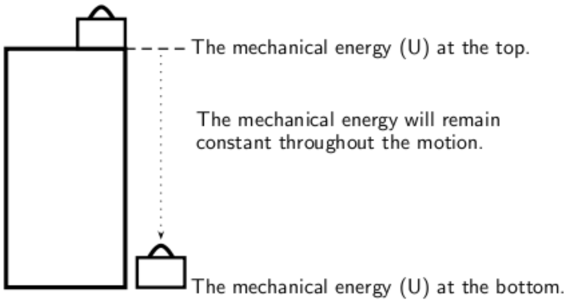
In the presence of internal forces like the force due to gravity or the force in a spring, mechanical energy is conserved.

The following simulation covers the law of conservation of energy.
run demo

Using the Law of Conservation of Energy

Mechanical energy is conserved (in the absence of friction). Therefore we can say that the sum of the PE and the KE anywhere during the motion must be equal to the sum of the PE and the KE anywhere else in the motion.

We can now apply this to the example of the suitcase on the cupboard. Consider the mechanical energy of the suitcase at the top and at the bottom. We can say:


The mechanical energy (U) at the top.
The mechanical energy will remain constant throughout the motion.
The mechanical energy (U) at the bottom.

Equation:

$$\begin{aligned}
U_{top} &= U_{bottom} \\
PE_{top} + KE_{top} &= PE_{bottom} + KE_{bottom} \\
mgh + \frac{1}{2}mv^2 &= mgh + \frac{1}{2}mv^2 \\
(1)(9,8)(2) + 0 &= 0 + \frac{1}{2}(1)\left(v^2\right) \\
19,6 \text{ J} &= \frac{1}{2}v^2 \\
39,2 &= v^2 \\
v &= 6,26\,\mathrm{m}\cdot\mathrm{s}^{-1}
\end{aligned}$$

The suitcase will strike the ground with a velocity of 6,26 $m \cdot s^{-1}$.

From this we see that when an object is lifted, like the suitcase in our example, it gains potential energy. As it falls back to the ground, it will lose this potential energy, but gain kinetic energy. We know that energy cannot be created or destroyed, but only changed from one form into another. In our example, the potential energy that the suitcase loses is changed to kinetic energy.

The suitcase will have maximum potential energy at the top of the cupboard and maximum kinetic energy at the bottom of the cupboard. Halfway down it will have half kinetic energy and half potential energy. As it moves down, the potential energy will be converted (changed) into kinetic energy until all the potential energy is gone and only kinetic energy is left. The 19,6 J of potential energy at the top will become 19,6 J of kinetic energy at the bottom.

Exercise:

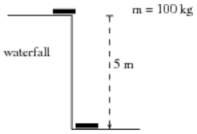
Using the Law of Conservation of Mechanical Energy

Problem:

During a flood a tree truck of mass $100\ \text{kg}$ falls down a waterfall. The waterfall is 5 m high. If air resistance is ignored, calculate

1. the potential energy of the tree trunk at the top of the waterfall.
2. the kinetic energy of the tree trunk at the bottom of the waterfall.

3. the magnitude of the velocity of the tree trunk at the bottom of the waterfall.



Solution:

Analyse the question to determine what information is provided

- The mass of the tree trunk $m = 100\ \text{kg}$
- The height of the waterfall $h = 5\,m$. These are all in SI units so we do not have to convert.

Analyse the question to determine what is being asked

- Potential energy at the top
- Kinetic energy at the bottom
- Velocity at the bottom

Calculate the potential energy.

Equation:

$$\begin{aligned} PE &= mgh \\ PE &= (100)(9,8)(5) \\ PE &= 4900\ \text{J} \end{aligned}$$

Calculate the kinetic energy.

The kinetic energy of the tree trunk at the bottom of the waterfall is equal to the potential energy it had at the top of the waterfall. Therefore $KE = 4\ 900\ J$.

Calculate the velocity.

To calculate the velocity of the tree trunk we need to use the equation for kinetic energy.

Equation:

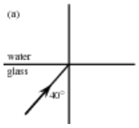
$$\begin{aligned} KE &= \frac{1}{2}mv^2 \\ 4900 &= \frac{1}{2}(100)\left(v^2\right) \\ 98 &= v^2 \\ v &= 9,899\ldots \\ v &= 9,90\,\text{m}\cdot\text{s}^{-1}\ \text{downwards} \end{aligned}$$

Exercise: Pendulum

Problem:

A $2\ \text{kg}$ metal ball is suspended from a rope. If it is released from point A and swings down to the point B (the bottom of its arc):

1. Show that the velocity of the ball is independent of it mass.
2. Calculate the velocity of the ball at point B.



Solution:

Analyse the question to determine what information is provided

- The mass of the metal ball is $m = 2\ \text{kg}$
- The change in height going from point A to point B is $h = 0{,}5\,m$
- The ball is released from point A so the velocity at point, $v_A = 0\,m \cdot s^{-1}$.

All quantities are in SI units.

Analyse the question to determine what is being asked

- Prove that the velocity is independent of mass.
- Find the velocity of the metal ball at point B.

Apply the Law of Conservation of Mechanical Energy to the situation

As there is no friction, mechanical energy is conserved. Therefore:

Equation:

$$\begin{aligned} U_A &= U_B \\ PE_A + KE_A &= PE_B + KE_B \\ mgh_A + \frac{1}{2}m(v_A)^2 &= mgh_B + \frac{1}{2}m(v_B)^2 \\ mgh_A + 0 &= 0 + \frac{1}{2}m(v_B)^2 \\ mgh_A &= \frac{1}{2}m(v_B)^2 \end{aligned}$$

As the mass of the ball m appears on both sides of the equation, it can be eliminated so that the equation becomes:

Equation:

$$gh_A = \frac{1}{2}(v_B)^2$$

Equation:

$$2gh_A = (v_B)$$

Calculate the velocity of the ball

We can use the equation above, or do the calculation from 'first principles':

Equation:

$$\begin{aligned} (v_B)^2 &= 2gh_A \\ (v_B)^2 &= (2)(9.8)(0,5) \\ (v_B)^2 &= 9,8 \\ v_B &= \sqrt{9,8}\ \mathrm{m \cdot s^{-1}} \end{aligned}$$

Potential Energy

1. A tennis ball, of mass $120\ g$, is dropped from a height of $5\ m$. Ignore air friction.

   a. What is the potential energy of the ball when it has fallen $3\ m$?
   b. What is the velocity of the ball when it hits the ground?

   Click here for the solution
2. A bullet, mass $50\ g$, is shot vertically up in the air with a muzzle velocity of $200\ m \cdot s^{-1}$. Use the Principle of Conservation of Mechanical Energy to determine the height that the bullet will reach. Ignore air friction.
   Click here for the solution
3. A skier, mass $50\ \text{kg}$, is at the top of a $6{,}4\,m$ ski slope.

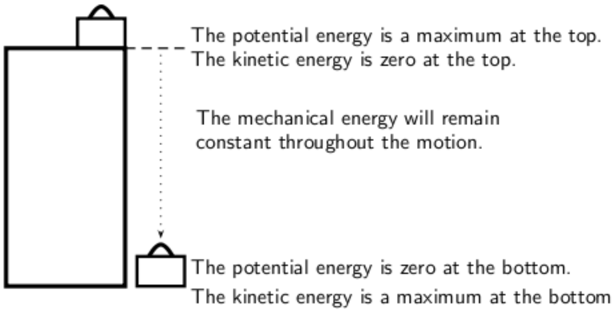
   a. Determine the maximum velocity that she can reach when she skies to the bottom of the slope.
   b. Do you think that she will reach this velocity? Why/Why not?

   Click here for the solution

4. A pendulum bob of mass $1{,}5\ \text{kg}$, swings from a height A to the bottom of its arc at B. The velocity of the bob at B is $4\,m \cdot s^{-1}$. Calculate the height A from which the bob was released. Ignore the effects of air friction.
   Click here for the solution
5. Prove that the velocity of an object, in free fall, in a closed system, is independent of its mass.
   Click here for the solution

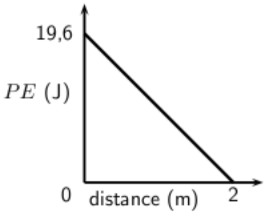
Energy graphs

Let us consider our example of the suitcase on the cupboard, once more.

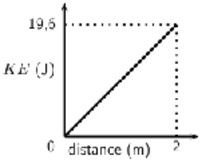

The potential energy is a maximum at the top.
The kinetic energy is zero at the top.
The mechanical energy will remain constant throughout the motion.
The potential energy is zero at the bottom.
The kinetic energy is a maximum at the bottom

Let's look at each of these quantities and draw a graph for each. We will look at how each quantity changes as the suitcase falls from the top to the bottom of the cupboard.

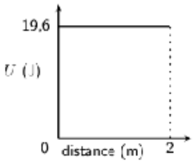
- **Potential energy**: The potential energy starts off at a maximum and decreases until it reaches zero at the bottom of the cupboard. It had fallen a distance of 2 metres.

- **Kinetic energy**: The kinetic energy is zero at the start of the fall. When the suitcase reaches the ground, the kinetic energy is a maximum. We also use distance on the x-axis.

- **Mechanical energy**: The mechanical energy is constant throughout the motion and is always a maximum. At any point in time, when we add the potential energy and the kinetic energy, we will get the same number.

The following presentation provides a summary of some of the concepts covered in this chapter.

[missing_resource: http://static.slidesharecdn.com/swf/ssplayer2.swf?doc=energy-100511053412-phpapp02&stripped_title=energy-4047773]

Summary

- Mass is the amount of matter an object is made up of.
- Weight is the force with which the Earth attracts a body towards its centre.
- Newtons Law of Gravitation.
- A body is in free fall if it is moving in the Earth's gravitational field and no other forces act on it.
- The equations of motion can be used for free fall problems. The acceleration (a) is equal to the acceleration due to gravity (g).
- The potential energy of an object is the energy the object has due to his position above a reference point.
- The kinetic energy of an object is the energy the object has due to its motion.
- Mechanical energy of an object is the sum of the potential energy and kinetic energy of the object.
- The unit for energy is the joule (J).
- The Law of Conservation of Energy states that energy cannot be created or destroyed, but can only be changed from one form into another.
- The Law of Conservation of Mechanical Energy states that the total mechanical energy of an isolated system remains constant.
- The table below summarises the most important equations:

Weight	$F_g = m \cdot g$
Equation of motion	$v_f = v_i + gt$
Equation of motion	$\Delta x = \frac{(v_i + v_f)}{2} t$
Equation of motion	$\Delta x = v_i t + \frac{1}{2} g t^2$
Equation of motion	$v_f^2 = v_i^2 + 2g\Delta x$
Potential Energy	$PE = mgh$
Kinetic Energy	$KE = \frac{1}{2} m v^2$
Mechanical Energy	$U = KE + PE$

End of Chapter Exercises: Gravity and Mechanical Energy

1. Give one word/term for the following descriptions.
   a. The force with which the Earth attracts a body.
   b. The unit for energy.
   c. The movement of a body in the Earth's gravitational field when no other forces act on it.
   d. The sum of the potential and kinetic energy of a body.
   e. The amount of matter an object is made up of.

   Click here for the solution
2. Consider the situation where an apple falls from a tree. Indicate whether the following statements regarding this situation are TRUE or FALSE. Write only 'true' or 'false'. If the statement is false, write down the correct statement.

   a. The potential energy of the apple is a maximum when the apple lands on the ground.
   b. The kinetic energy remains constant throughout the motion.
   c. To calculate the potential energy of the apple we need the mass of the apple and the height of the tree.
   d. The mechanical energy is a maximum only at the beginning of the motion.
   e. The apple falls at an acceleration of $9{,}8\ m \cdot s^{-2}$.

   Click here for the solution

3. [IEB 2005/11 HG] Consider a ball dropped from a height of $1\ m$ on Earth and an identical ball dropped from $1\ m$ on the Moon. Assume both balls fall freely. The acceleration due to gravity on the Moon is one sixth that on Earth. In what way do the following compare when the ball is dropped on Earth and on the Moon.

	Mass	**Weight**	**Increase in kinetic energy**
(a)	the same	the same	the same
(b)	the same	greater on Earth	greater on Earth
(c)	the same	greater on Earth	the same
(d)	greater on Earth	greater on Earth	greater on Earth

   Click here for the solution

4. A man fires a rock out of a slingshot directly upward. The rock has an initial velocity of $15\ m \cdot s^{-1}$.

   a. How long will it take for the rock to reach its highest point?
   b. What is the maximum height that the rock will reach?
   c. Draw graphs to show how the potential energy, kinetic energy and mechanical energy of the rock changes as it moves to its highest point.

   Click here for the solution

5. A metal ball of mass $200\ g$ is tied to a light string to make a pendulum. The ball is pulled to the side to a height (A), $10\ \text{cm}$ above the lowest point of the swing (B). Air friction and the mass of the string can be ignored. The ball is let go to swing freely.

   a. Calculate the potential energy of the ball at point A.
   b. Calculate the kinetic energy of the ball at point B.
   c. What is the maximum velocity that the ball will reach during its motion?

   Click here for the solution

6. A truck of mass $1{,}2\ \text{tons}$ is parked at the top of a hill, $150\ m$ high. The truck driver lets the truck run freely down the hill to the bottom.

   a. What is the maximum velocity that the truck can achieve at the bottom of the hill?
   b. Will the truck achieve this velocity? Why/why not?

   Click here for the solution

7. A stone is dropped from a window, $3\ m$ above the ground. The mass of the stone is $25\ g$.

   a. Use the Equations of Motion to calculate the velocity of the stone as it reaches the ground.
   b. Use the Principle of Conservation of Energy to prove that your answer in (a) is correct.

   Click here for the solution

2D and 3D Wavefronts

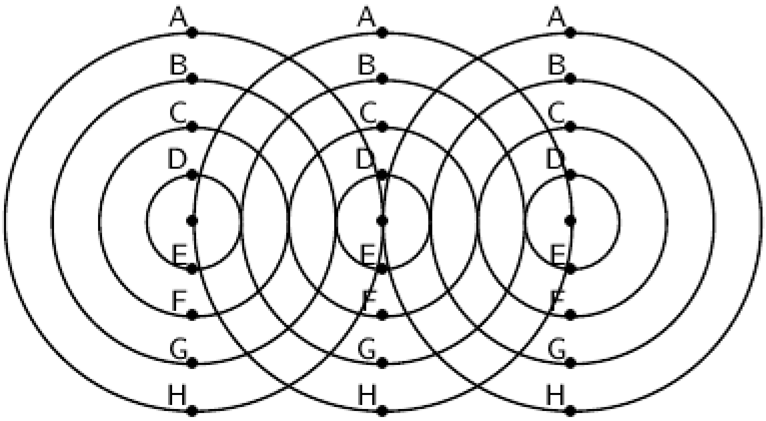
Introduction

You have learnt about the basic principles of reflection and refraction. In this chapter, you will learn about phenomena that arise with waves in two and three dimensions: interference and diffraction.

Wavefronts

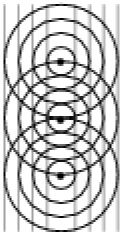
Investigation : Wavefronts

The diagram shows three identical waves being emitted by three point sources. All points marked with the same letter are in phase. Join all points with the same letter.



What type of lines (straight, curved, etc) do you get? How does this compare to the line that joins the sources?

Consider three point sources of waves. If each source emits waves isotropically (i.e. the same in all directions) we will get the situation shown in as shown in [link].



Wavefronts are imaginary lines joining waves that are in phase. In the example, the wavefronts (shown by the grey, vertical lines) join all waves at the crest of their cycle.

We define a **wavefront** as the imaginary line that joins waves that are in phase. These are indicated by the grey, vertical lines in [link]. The points that are in phase can be peaks, troughs or anything in between, it doesn't matter which points you choose as long as they are in phase.

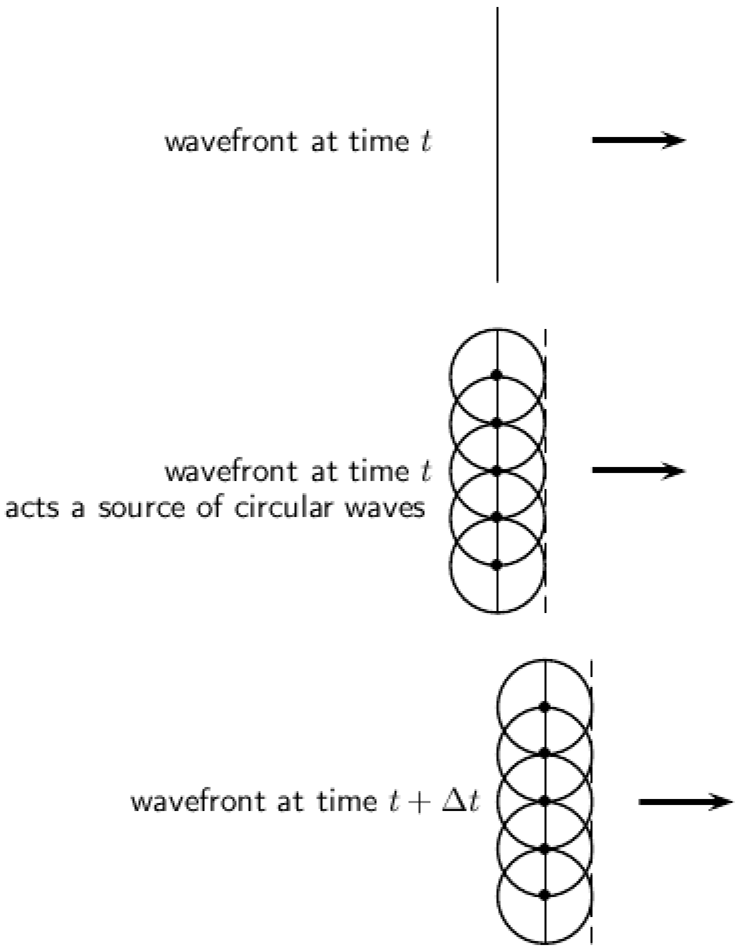
The Huygens Principle

Christiaan Huygens described how to determine the path of waves through a medium.

The Huygens Principle

> Each point on a wavefront acts like a point source of circular waves. The waves emitted from these point sources interfere to form another wavefront.

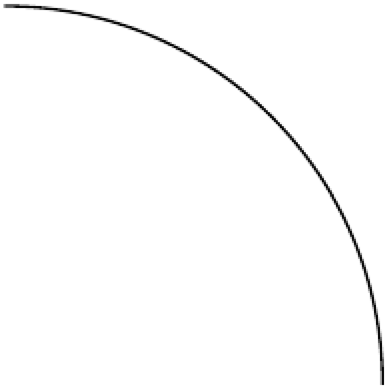
A simple example of the Huygens Principle is to consider the single wavefront in [link].


wavefront at time t
wavefront at time t
acts a source of circular waves
wavefront at time $t + \Delta t$

A single wavefront at time t acts as a series of point sources of circular waves that interfere to give a new wavefront at a time $t + \Delta t$. The process continues and applies to any shape of waveform.

Exercise:
Application of the Huygens Principle

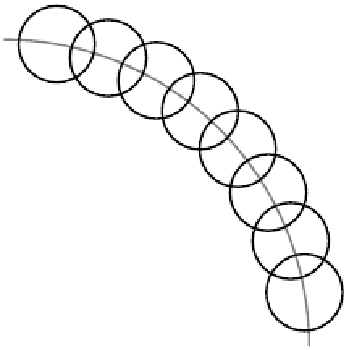
Problem: Given the wavefront,



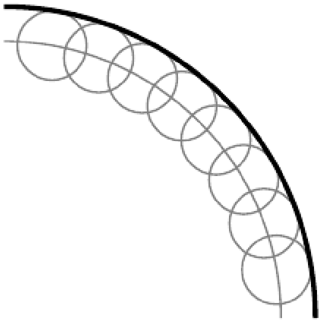
use the Huygens Principle to determine the wavefront at a later time.

Solution:

Draw circles at various points along the given wavefront



Join the crests of each circle to get the wavefront at a later time



Note:
Interesting Fact
Christiaan Huygens (14 April 1629 - 8 July 1695), was a Dutch mathematician, astronomer and physicist; born in The Hague as the son of Constantijn Huygens. He studied law at the University of Leiden and the College of Orange in Breda before turning to science. Historians commonly associate Huygens with the scientific revolution.

Huygens generally receives minor credit for his role in the development of modern calculus. He also achieved note for his arguments that light consisted of waves; see: wave-particle duality in Chapter [link]. In 1655, he discovered Saturn's moon Titan. He also examined Saturn's planetary rings, and in 1656 he discovered that those rings consisted of rocks. In the same year he observed and sketched the Orion Nebula. He also discovered several interstellar nebulae and some double stars.

Interference

Interference occurs when two identical waves pass through the same region of space at the same time resulting in a superposition of waves. There are two types of interference which is of interest: **constructive** interference and **destructive** interference.

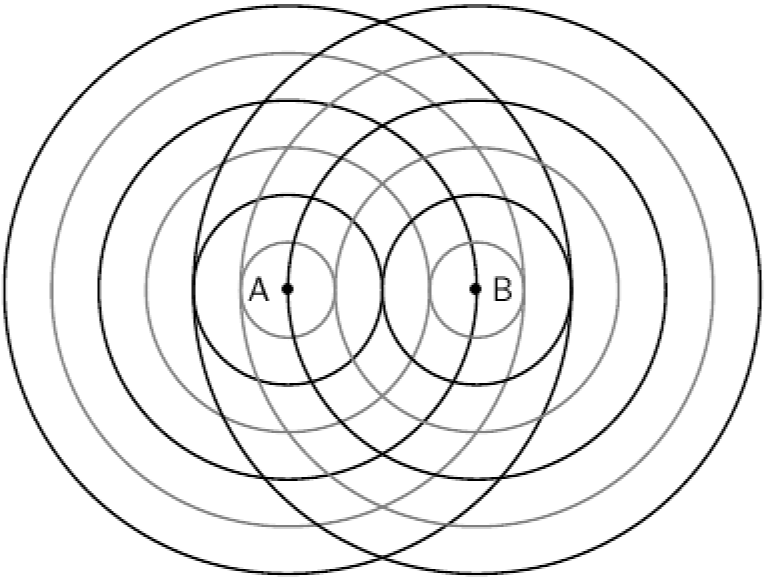
Constructive interference occurs when both waves have a displacement in the same direction, while destructive interference occurs when one wave has a displacement in the opposite direction to the other, thereby resulting in a cancellation. When two waves interfere destructively, the resultant absolute displacement of the medium is less than in either of the individual displacements. When *total* destructive interference occurs, there is no displacement of the medium. For constructive interference the displacement of the medium is greater than the individual displacements.

Constructive interference is the result of two waves with similar phase overlapping. This means that positive parts of one wave tend to overlap with positive parts of the other, and alike for the negative parts. When positive is added to positive and negative is added to negative, the net absolute displacement of the medium is greater than the displacements of the individual waves.

Destructive interference, on the other hand, is the result of two waves with non-similar phase (i.e. anti-phase) overlapping. This means that the positive parts of one wave tend to align with the negative parts of the second wave. When the waves are added together, the positive and negative contributions lead to a net absolute displacement of the medium which is less than the absolute displacements of either of the individual waves. A place where destructive interference takes place is called a node.

Waves can interfere at places where there is never a trough and trough or peak and peak or trough and peak at the same time. At these places the waves will add together and the resultant displacement will be the sum of the two waves but they won't be points of maximum interference.

Consider the two identical waves shown in the picture below. The wavefronts of the peaks are shown as black lines while the wavefronts of the troughs are shown as grey lines. You can see that the black lines cross other black lines in many places. This means two peaks are in the same place at the same time so we will have constructive interference where the two peaks add together to form a bigger peak.



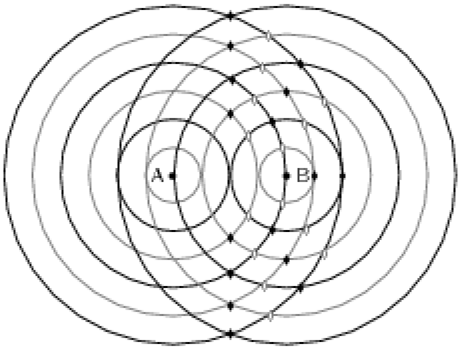
Two points sources (A and B) radiate identical waves. The wavefronts of the peaks (black lines) and troughs (grey lines) are shown. Constructive interference occurs where two black lines intersect or where two gray lines intersect. Destructive interference occurs where a black line intersects with a grey line.

When the grey lines cross other grey lines there are two troughs in the same place at the same time so we will have constructive interference where the two troughs add together to form a bigger trough.

In the case where a grey line crosses a black line we are seeing a trough and peak at the same time. These will cancel each other out and the medium will have no displacement at that point.

- black line + black line = peak + peak = constructive interference
- grey line + grey line = trough + trough = constructive interference
- black line + grey line = grey line + black line = peak + trough = trough + peak = destructive interference

On half the picture below, we have marked the constructive interference with a solid black diamond and the destructive interference with a hollow diamond.



To see if you understand it, cover up the half we have marked with diamonds and try to work out which points are constructive and destructive on the other half of the picture. The two halves are mirror images of each other so you can check yourself.

Diffraction

One of the most interesting, and also very useful, properties of waves is **diffraction**.

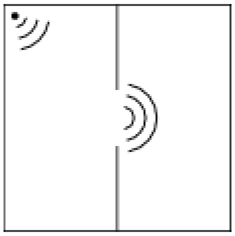
Diffraction

> Diffraction is the ability of a wave to spread out in wavefronts as the wave passes through a small aperture or around a sharp edge.

Diffraction

Diffraction refers to various phenomena associated with wave propagation, such as the bending, spreading and interference of waves emerging from an aperture. It occurs with any type of wave, including sound waves, water waves, electromagnetic waves such as light and radio waves. While diffraction always occurs, its effects are generally only noticeable for waves where the wavelength is on the order of the feature size of the diffracting objects or apertures.

For example, if two rooms are connected by an open doorway and a sound is produced in a remote corner of one of them, a person in the other room will hear the sound as if it originated at the doorway.



As far as the second room is concerned, the vibrating air in the doorway is the source of the sound. The same is true of light passing the edge of an obstacle, but this is not as easily observed because of the short wavelength of visible light.

This means that when waves move through small holes they appear to bend around the sides because there are not enough points on the wavefront to form another straight wavefront. This is bending round the sides we call *diffraction*.

Diffraction

Diffraction effects are more clear for water waves with longer wavelengths. Diffraction can be demonstrated by placing small barriers and obstacles in a ripple tank and observing the path of the water waves as they encounter the obstacles. The waves are seen to pass around the barrier into the regions behind it; subsequently the water behind the barrier is disturbed. The amount of diffraction (the sharpness of the bending) increases with increasing wavelength and decreases with decreasing wavelength. In fact, when the wavelength of the waves are smaller than the obstacle, no noticeable diffraction occurs.

Experiment : Diffraction

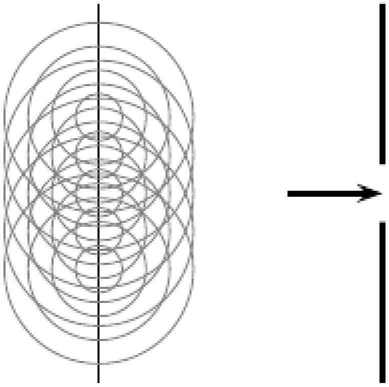
Water waves in a ripple tank can be used to demonstrate diffraction and interference.

- Turn on the wave generator so that it produces waves with a high frequency (short wavelength).
    - Place a few obstacles, one at a time, (e.g. a brick or a ruler) in the ripple tank. What happens to the wavefronts as they propagate near/past the obstacles? Draw your observations.
    - How does the diffraction change when you change the size of the object?
- Now turn down the frequency of the wave generator so that it produces waves with longer wavelengths.
    - Place the same obstacles in the ripple tank (one at a time). What happens to the wavefronts as they propagate near/past the obstacles? Draw your observations.
    - How does the diffraction change from the higher frequency case?
- Remove all obstacles from the ripple tank and insert a second wave generator. Turn on both generators so that they start at the same time and have the same frequency.
    - What do you notice when the two sets of wavefronts meet each other?
    - Can you identify regions of constructive and destructive interference?
- Now turn on the generators so that they are out of phase (i.e. start them so that they do not make waves at exactly the same time).
    - What do you notice when the two sets of wavefronts meet each other?
    - Can you identify regions of constructive and destructive interference?

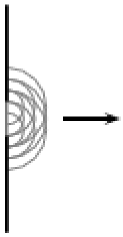
Diffraction through a Slit

When a wave strikes a barrier with a hole only part of the wave can move through the hole. If the hole is similar in size to the wavelength of the wave then diffraction occurs. The waves that come through the hole no longer looks like a straight wave front. It bends around the edges of the hole. If the hole is small enough it acts like a point source of circular waves.

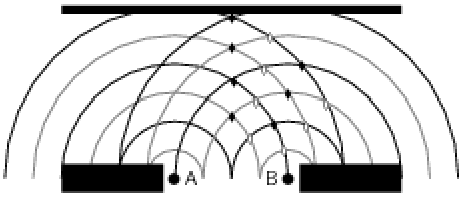
Now if we allow the wavefront to impinge on a barrier with a hole in it, then only the points on the wavefront that move into the hole can continue emitting forward moving waves - but because a lot of the wavefront has been removed, the points on the edges of the hole emit waves that bend round the edges.



If you employ Huygens' principle you can see the effect is that the wavefronts are no longer straight lines.

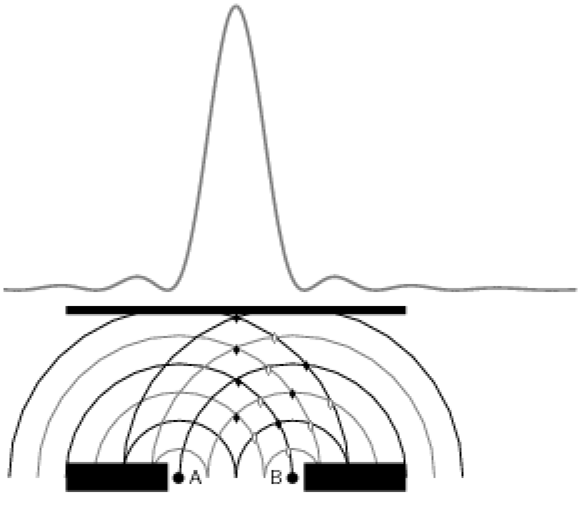


Each point of the slit acts like a point source. If we think about the two point sources on the edges of the slit and call them A and B then we can go back to the diagram we had earlier but with some parts blocked by the wall.

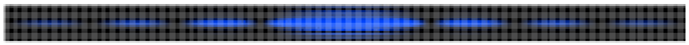


If this diagram were showing sound waves then the sound would be louder (constructive interference) in some places and quieter (destructive interference) in others. You can start to see that there will be a pattern (interference pattern) to the louder and quieter places. If we were studying light waves then the light would be brighter in some places than others depending on the interference.

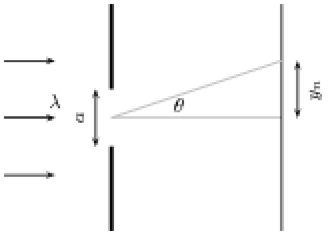
The intensity (how bright or loud) of the interference pattern for a single narrow slit looks like this:



The picture above shows how the waves add together to form the interference pattern. The peaks correspond to places where the waves are adding most intensely and the minima are places where destructive interference is taking place. When looking at interference patterns from light the spectrum looks like:



There is a formula we can use to determine where the peaks and minima are in the interference spectrum. There will be more than one minimum. There are the same number of minima on either side of the central peak and the distances from the first one on each side are the same to the peak. The distances to the peak from the second minimum on each side is also the same, in fact the two sides are mirror images of each other. We label the first minimum that corresponds to a positive angle from the centre as $m = 1$ and the first on the other side (a negative angle from the centre) as $m = -1$, the second set of minima are labelled $m = 2$ and $m = -2$ etc.



The equation for the angle at which the minima occur is given in the definition below:

Interference Minima

The angle at which the minima in the interference spectrum occur is:
Equation:

$$\sin\theta = \frac{m\lambda}{a}$$

where

θ is the angle to the minimum

a is the width of the slit

λ is the wavelength of the impinging wavefronts

m is the order of the mimimum, $m = \pm 1, \pm 2, \pm 3, \ldots$

Exercise: Diffraction Minimum I

Problem:

A slit with a width of 2511 nm has red light of wavelength 650 nm impinge on it. The diffracted light interferes on a surface. At which angle will the first minimum be?

Solution:

Check what you are given We know that we are dealing with interference patterns from the diffraction of light passing through a slit. The slit has a width of 2511 nm which is 2511×10^{-9} m and we know that the wavelength of the light is 650 nm which is 650×10^{-9} m. We are looking to determine the angle to first minimum so we know that $m = 1$.

Applicable principles We know that there is a relationship between the slit width, wavelength and interference minimum angles:

Equation:

$$\sin\theta = \frac{m\lambda}{a}$$

We can use this relationship to find the angle to the minimum by substituting what we know and solving for the angle.

SubstitutionEquation:

The first minimum is at 15 degrees from the centre peak.

$$\begin{aligned}
\sin\theta &= \frac{650\times10^{-9}\text{m}}{2511\times10^{-9}\text{m}}\\
\sin\theta &= \frac{650}{2511}\\
\sin\theta &= 0.258861012\\
\theta &= \sin^{-1}0.258861012\\
\theta &= 15^{o}
\end{aligned}$$

Exercise:
Diffraction Minimum II

Problem:

A slit with a width of 2511 nm has green light of wavelength 532 nm impinge on it. The diffracted light interferers on a surface, at what angle will the first minimum be?

Solution:

Check what you are given We know that we are dealing with interference patterns from the diffraction of light passing through a slit. The slit has a width of 2511 nm which is 2511×10^{-9} m and we know that the wavelength of the light is 532 nm which is 532×10^{-9} m. We are looking to determine the angle to first minimum so we know that $m=1$.

Applicable principles We know that there is a relationship between the slit width, wavelength and interference minimum angles:

Equation:

$$\sin\theta = \frac{m\lambda}{a}$$

We can use this relationship to find the angle to the minimum by substituting what we know and solving for the angle.

SubstitutionEquation:

The first minimum is at 12.2 degrees from the centre peak.

$$\begin{aligned}\sin\theta &= \frac{532\times10^{-9}\text{m}}{2511\times10^{-9}\text{m}}\\ \sin\theta &= \frac{532}{2511}\\ \sin\theta &= 0.211867782\\ \theta &= \sin^{-1}0.211867782\\ \theta &= 12.2^{o}\end{aligned}$$

From the formula $\sin\theta=\frac{m\lambda}{a}$ you can see that a smaller wavelength for the same slit results in a smaller angle to the interference minimum. This is something you just saw in the two worked examples. Do a sanity check, go back and see if the answer makes sense. Ask yourself which light had the longer wavelength, which light had the larger angle and what do you expect for longer wavelengths from the formula.

Exercise:

Diffraction Minimum III

Problem:

A slit has a width which is unknown and has green light of wavelength 532 nm impinge on it. The diffracted light interferers on a surface, and the first minimum is measure at an angle of 20.77 degrees?

Solution:

Check what you are given We know that we are dealing with interference patterns from the diffraction of light passing through a slit. We know that the wavelength of the light is 532 nm which is 532×10^{-9} m. We know the angle to first minimum so we know that $m=1$ and $\theta=20.77^{\text{o}}$.

Applicable principles We know that there is a relationship between the slit width,

Equation:

$$\sin\theta=\frac{m\lambda}{a}$$

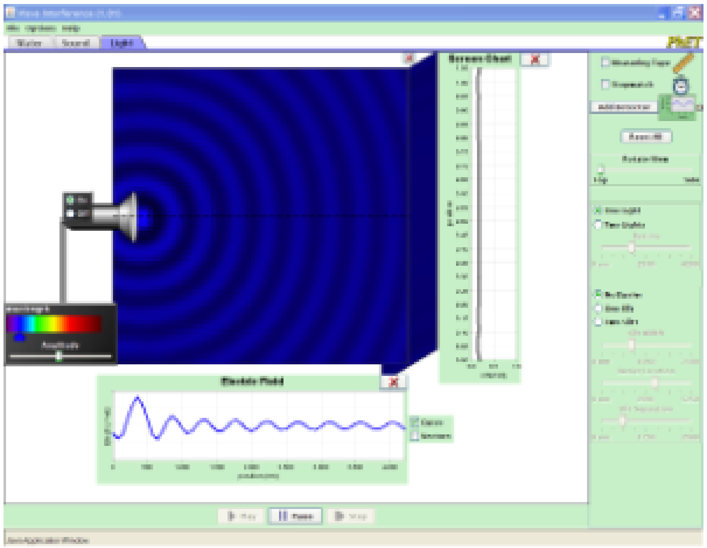
We can use this relationship to find the width by wavelength and interference minimum angles:

substituting what we know and solving for the width.

SubstitutionEquation:

$$\begin{aligned}
\sin\theta &= \frac{532\times10^{-9}\ \text{m}}{a}\\
\sin 20.77^{o} &= \frac{532\times10^{-9}}{a}\\
a &= \frac{532\times10^{-9}}{0.354666667}\\
a &= 1500\times10^{-9}\\
a &= 1500\ \ \text{nm}
\end{aligned}$$

The slit width is 1500 nm.



[run demo]

Shock Waves and Sonic Booms

Now we know that the waves move away from the source at the speed of sound. What happens if the source moves at the same time as emitting sounds? Once a sound wave has been emitted it is no longer connected to the source so if the source moves it doesn't change the way the sound wave is propagating through the medium. This means a source can actually catch up to the sound waves it has emitted.

The speed of sound is very fast in air, about $340\ \text{m}\cdot\text{s}^{-1}$, so if we want to talk about a source catching up to sound waves then the source has to be able to move very fast. A good source of sound waves to discuss is a jet aircraft. Fighter jets can move very fast and they are very noisy so they are a good source of sound for our discussion. Here are the speeds for a selection of aircraft that can fly faster than the speed of sound.

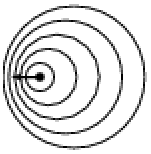
Aircraft	speed at altitude ($\text{km}\cdot\text{h}^{-1}$)	speed at altitude ($\text{m}\cdot\text{s}^{-1}$)
Concorde	2 330	647
Gripen	2 410	669
Mirage F1	2 573	715
Mig 27	1 885	524
F 15	2 660	739
F 16	2 414	671

Subsonic Flight

Subsonic

> Subsonic refers to speeds slower than the speed of sound.

When a source emits sound waves and is moving slower than the speed of sound you get the situation in this picture. Notice that the source moving means that the wavefronts, and therefore peaks in the wave, are actually closer together in the one direction and further apart in the other.



subsonic flight

If you measure the waves on the side where the peaks are closer together you'll measure a different wavelength than on the other side of the source. This means that the noise from the source will sound different on the different sides. This is called the *Doppler Effect*.

Doppler Effect

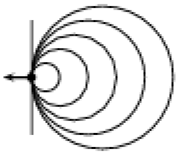
> when the wavelength and frequency measured by an observer are different to those emitted by the source due to movement of the source or observer.

Supersonic Flight

Supersonic

> Supersonic refers to speeds faster than the speed of sound.

If a plane flies at exactly the speed of sound then the waves that it emits in the direction it is flying won't be able to get away from the plane. It also means that the next sound wave emitted will be exactly on top of the previous one, look at this picture to see what the wavefronts would look like:



shock wave at Mach 1

Sometimes we use the speed of sound as a reference to describe the speed of the object (aircraft in our discussion).

Mach Number

> The Mach Number is the ratio of the speed of an object to the speed of sound in the surrounding medium.

Mach number is tells you how many times faster than sound the aircraft is moving.

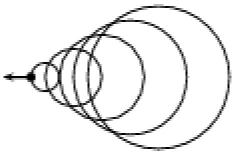
- Mach Number $<$ 1 : aircraft moving slower than the speed of sound
- Mach Number $=$ 1 : aircraft moving at the speed of sound
- Mach Number $>$ 1 : aircraft moving faster than the speed of sound

To work out the Mach Number divide the speed of the aircraft by the speed of sound.
Equation:

$$\text{Mach Number} = \frac{\mathrm{v}_{\text{aircraft}}}{\mathrm{v}_{\text{sound}}}$$

Remember: the units must be the same before you divide.

If the aircraft is moving faster than the speed of sound then the wavefronts look like this:



supersonic shock wave

If the source moves faster than the speed of sound, a cone of wave fronts is created. This is called a Mach cone. From constructive interference, we know that two peaks that add together form a larger peak. In a Mach cone many, many peaks add together to form a very large peak. This is a sound wave so the large peak is a very, very loud sound wave. This sounds like a huge "boom" and we call the noise a *sonic boom*.

Sonic Boom

> A sonic boom is the sound heard by an observer as a shockwave passes.

Exercise:
Mach Speed I

Problem:

An aircraft flies at 1300 $\mathrm{km \cdot h^{-1}}$ and the speed of sound in air is $340\ \mathrm{m \cdot s^{-1}}$. What is the Mach Number of the aircraft?

Solution:

Check what you are given We know we are dealing with Mach Number. We are given the speed of sound in air, $340\ \mathrm{m \cdot s^{-1}}$, and the speed of the aircraft, 1300 $\mathrm{km \cdot h^{-1}}$. The speed of the aircraft is in different units to the speed of sound so we need to convert the units:

Equation:

$$\begin{aligned}1300\mathrm{km \cdot h^{-1}} &= 1300\mathrm{km \cdot h^{-1}}\\ 1300\mathrm{km \cdot h^{-1}} &= 1300 \times \frac{1000\mathrm{m}}{3600\mathrm{s}}\\ 1300\mathrm{km \cdot h^{-1}} &= 361.1\ \mathrm{m \cdot s^{-1}}\end{aligned}$$

Applicable principles	We know that there is a relationship between the Mach Number, the speed of sound and the speed of the aircraft:	**Equation:** $\text{Mach Number} = \frac{\text{v}_{\text{aircraft}}}{\text{v}_{\text{sound}}}$	We can use this relationship to find the Mach Number.
Substitution	**Equation:** $\text{Mach Number} = \frac{v_{aircraft}}{v_{sound}}$ $\text{Mach Number} = \frac{361.1}{340}$ $\text{Mach Number} = 1.06$		The Mach Number is 1.06.

Mach Number

In this exercise we will determine the Mach Number for the different aircraft in the previous table. To help you get started we have calculated the Mach Number for the Concord with a speed of sound $v_{sound} = 340 \text{ ms}^{-1}$.

For the Condorde we know the speed and we know that:

Equation:

$$\text{Mach Number} = \frac{\text{v}_{\text{aircraft}}}{\text{v}_{\text{sound}}}$$

For the Concorde this means that

Equation:

$$\begin{aligned}\text{Mach Number} &= \frac{647}{340}\\ &= 1.9\end{aligned}$$

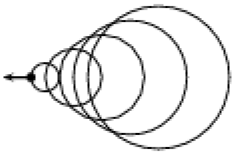
Aircraft	speed at altitude ($\mathrm{km \cdot h^{-1}}$)	speed at altitude ($\mathrm{m \cdot s^{-1}}$)	Mach Number
Concorde	2 330	647	1.9
Gripen	2 410	669	
Mirage F1	2 573	715	
Mig 27	1 885	524	
F 15	2 660	739	
F 16	2 414	671	

Now calculate the Mach Numbers for the other aircraft in the table.

Mach Cone

The shape of the Mach Cone depends on the speed of the aircraft. When the Mach Number is 1 there is no cone but as the aircraft goes faster and faster the angle of the cone gets smaller and smaller.

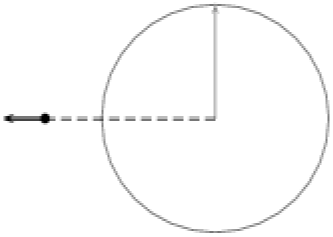
If we go back to the supersonic picture we can work out what the angle of the cone must be.



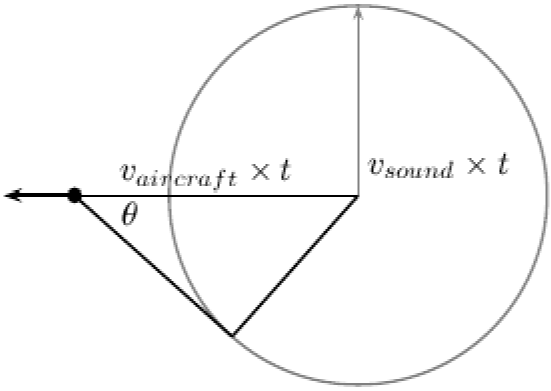
supersonic shock wave

We build a triangle between how far the plane has moved and how far a wavefront at right angles to the direction the plane is flying has moved:

An aircraft emits a sound wavefront. The wavefront moves at the speed of sound 340 $\text{m} \cdot \text{s}^{-1}$ and the aircraft moves at Mach 1.5, which is $1.5 \times 340 = 510 \text{ m} \cdot \text{s}^{-1}$. The aircraft travels faster than the wavefront. If we let the wavefront travel for a time t then the following diagram will apply:



We know how fast the wavefront and the aircraft are moving so we know the distances that they have traveled:



The angle between the cone that forms and the direction of the plane can be found from the right-angle triangle we have drawn into the figure. We know that $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ which in this figure means:

Equation:

$$\begin{aligned}\sin\theta &= \frac{\text{opposite}}{\text{hypotenuse}}\\ \sin\theta &= \frac{v_{sound} \times t}{v_{aircraft} \times t}\\ \sin\theta &= \frac{v_{sound}}{v_{aircraft}}\end{aligned}$$

In this case we have used sound and aircraft but a more general way of saying this is:

- aircraft = source
- sound = wavefront

We often just write the equation as:

Equation:

$$\begin{aligned}\sin\theta &= \frac{v_{sound}}{v_{aircraft}}\\ v_{aircraft}\sin\theta &= v_{sound}\\ v_{source}\sin\theta &= v_{wavefront}\\ v_s\sin\theta &= v_w\end{aligned}$$

From this equation, we can see that the faster the source (aircraft) moves, the smaller the angle of the Mach cone.

Mach Cone

In this exercise we will determine the Mach Cone Angle for the different aircraft in the table mentioned above. To help you get started we have calculated the Mach Cone Angle for the Concorde with a speed of sound $v_{sound} = 340\ \mathrm{m \cdot s^{-1}}$.

For the Condorde we know the speed and we know that:
Equation:

$$\sin\theta = \frac{v_{sound}}{v_{aircraft}}$$

For the Concorde this means that
Equation:

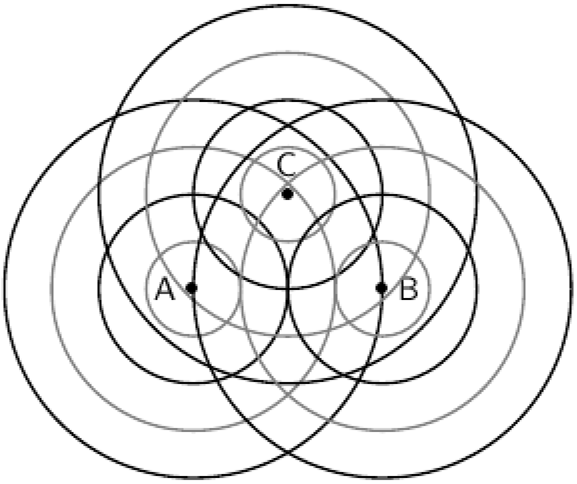
$$\begin{aligned}\sin\theta &= \frac{340}{647}\\ \theta &= \sin^{-1}\frac{340}{647}\\ \theta &= 31.7^{\mathrm{o}}\end{aligned}$$

Aircraft	speed at altitude ($\text{km} \cdot \text{h}^{-1}$)	speed at altitude ($\text{m} \cdot \text{s}^{-1}$)	Mach Cone Angle (degrees)
Concorde	2 330	647	31.7
Gripen	2 410	669	
Mirage F1	2 573	715	
Mig 27	1 885	524	
F 15	2 660	739	
F 16	2 414	671	

Now calculate the Mach Cone Angles for the other aircraft in the table.

End of Chapter Exercises

1. In the diagram below the peaks of wavefronts are shown by black lines and the troughs by grey lines. Mark all the points where constructive interference between two waves is taking place and where destructive interference is taking place. Also note whether the interference results in a peak or a trough.



2. For a slit of width 1300 nm, calculate the first 3 minima for light of the following wavelengths:

    1. blue at 475 nm
    2. green at 510 nm
    3. yellow at 570 nm
    4. red at 650 nm

3. For light of wavelength 540 nm, determine what the width of the slit needs to be to have the first minimum at:

    1. 7.76 degrees
    2. 12.47 degrees
    3. 21.1 degrees

4. For light of wavelength 635 nm, determine what the width of the slit needs to be to have the second minimum at:

    1. 12.22 degrees
    2. 18.51 degrees
    3. 30.53 degrees

5. If the first minimum is at 8.21 degrees and the second minimum is at 16.6 degrees, what is the wavelength of light and the width of the slit? (**Hint:** solve simultaneously.)
6. Determine the Mach Number, with a speed of sound of 340 $\text{m} \cdot \text{s}^{-1}$, for the following aircraft speeds:

    1. 640 $\mathrm{m \cdot s^{-1}}$
    2. 980 $\mathrm{m \cdot s^{-1}}$
    3. 500 $\mathrm{m \cdot s^{-1}}$
    4. 450 $\mathrm{m \cdot s^{-1}}$
    5. 1300 $\mathrm{km \cdot h^{-1}}$
    6. 1450 $\mathrm{km \cdot h^{-1}}$
    7. 1760 $\mathrm{km \cdot h^{-1}}$

7. If an aircraft has a Mach Number of 3.3 and the speed of sound is 340 $\mathrm{m \cdot s^{-1}}$, what is its speed?
8. Determine the Mach Cone angle, with a speed of sound of 340 $\mathrm{m \cdot s^{-1}}$, for the following aircraft speeds:

    1. 640 $\mathrm{m \cdot s^{-1}}$
    2. 980 $\mathrm{m \cdot s^{-1}}$
    3. 500 $\mathrm{m \cdot s^{-1}}$
    4. 450 $\mathrm{m \cdot s^{-1}}$
    5. 1300 $\mathrm{km \cdot h^{-1}}$
    6. 1450 $\mathrm{km \cdot h^{-1}}$
    7. 1760 $\mathrm{km \cdot h^{-1}}$

9. Determine the aircraft speed, with a speed of sound of 340 $\mathrm{m \cdot s^{-1}}$, for the following Mach Cone Angles:

    1. 58.21 degrees
    2. 49.07 degrees
    3. 45.1 degrees
    4. 39.46 degrees
    5. 31.54 degrees

Colour

Introduction

The light that human beings can see is called *visible light*. Visible light is actually just a small part of the large spectrum of electromagnetic radiation which you will learn more about in [link]. We can think of electromagnetic radiation and visible light as transverse waves. We know that transverse waves can be described by their amplitude, frequency (or wavelength) and velocity. The velocity of a wave is given by the product of its frequency and wavelength:

Equation:

$$v = f \times \lambda$$

However, electromagnetic radiation, including visible light, is special because, no matter what the frequency, it all moves at a **constant velocity** (in vacuum) which is known as the speed of light. The speed of light has the symbol c and is:

Equation:

$$c \;=\; 3 \times 10^{8} \;\; \mathrm{m.s^{-1}}$$

Since the *speed of light* is c, we can then say:

Equation:

$$c = f \times \lambda$$

Colour and Light

Our eyes are sensitive to visible light over a range of wavelengths from 390 nm to 780 nm (1 nm = 1×10^{-9} m). The different **colours** of light we see are related to specific *frequencies* (and *wavelengths*) of visible light. The wavelengths and frequencies are listed in [link].

Colour	Wavelength range (nm)	Frequency range (Hz)
violet	390 - 455	769 - 659 $\times 10^{12}$
blue	455 - 492	659 - 610 $\times 10^{12}$
green	492 - 577	610 - 520 $\times 10^{12}$

yellow	577 - 597	520 - 503 $\times 10^{12}$
orange	597 - 622	503 - 482 $\times 10^{12}$
red	622 - 780	482 - 385 $\times 10^{12}$

Colours, wavelengths and frequencies of light in the visible spectrum.

You can see from [link] that **violet** light has the *shortest wavelengths* and *highest frequencies* while **red** light has the *longest wavelengths* and *lowest frequencies*.

Exercise:

Calculating the frequency of light given the wavelength

Problem: A streetlight emits light with a wavelength of 520 nm.

1. What colour is the light? (Use [link] to determine the colour)
2. What is the frequency of the light?

Solution:

What is being asked and what information are we given?

We need to determine the colour and frequency of light with a wavelength of $\lambda = 520$ nm = 520×10^{-9} m.

Compare the wavelength of the light to those given in

We see from [link] that light with wavelengths between 492 - 577 nm is green. 520 nm falls into this range, therefore the colour of the light is green.

Next we need to calculate the frequency of the light

Equation:

We know that

$$c = f \times \lambda$$

We know c and we are given that $\lambda = 520 \times 10^{-9}$ m. So we can substitute in these values and solve for the frequency f. (**NOTE:** Don't forget to always change units into S.I. units! 1 nm = 1×10^{-9} m.)

Equation:

$$\begin{aligned} f &= \frac{c}{\lambda} \\ &= \frac{3 \times 10^{8}}{520 \times 10^{-9}} \\ &= 577 \times 10^{12} \text{ Hz} \end{aligned}$$

The frequency of the green light is 577×10^{12} Hz

Exercise:

Calculating the wavelength of light given the frequency

Problem: A streetlight also emits light with a frequency of 490×10^{12} Hz.

1. What colour is the light? (Use [link] to determine the colour)
2. What is the wavelength of the light?

Solution:

What is being asked and what information are we given?

We need to find the colour and wavelength of light which has a frequency of 490×10^{12} Hz and which is emitted by the streetlight.

Compare the wavelength of the light to those given in

We can see from [link] that orange light has frequencies between 503 - 482×10^{12} Hz. The light from the streetlight has $f = 490 \times 10^{12}$ Hz which fits into this range. Therefore the light must be orange in colour.

Next we need to calculate the wavelength of the light

Equation:

We know that

$$c = f \times \lambda$$

Equation:

We know

$$c = 3 \times 10^{8} \ \text{m.s}^{-1}$$

and we are given that $f = 490 \times 10^{12}$ Hz. So we can substitute in these values and solve for the wavelength λ.

$$\begin{aligned}
\lambda &= \frac{c}{f} \\
&= \frac{3 \times 10^{8}}{490 \times 10^{12}} \\
&= 6.122 \times 10 - 7 \ \text{m} \\
&= 612 \times 10^{-9} \ \text{m} \\
&= 612 \ \text{nm}
\end{aligned}$$

Therefore the orange light has a wavelength of 612 nm.

Exercise:
Frequency of Green

Problem:

The wavelength of green light ranges between 500 nm an d 565 nm. Calculate the range of frequencies that correspond to this range of wavelengths.

Solution:

Determine how to approach the problem

Equation:

Use

$$c = f \times \lambda$$

to determine f.

Calculate frequency corresponding to upper limit of wavelength range

Equation:

$$\begin{aligned} c &= f \times \lambda \\ f &= \frac{c}{\lambda} \\ &= \frac{3 \times 10^{8}\,\mathrm{m \cdot s^{-1}}}{565 \times 10^{-9}\,\mathrm{m}} \\ &= 5,31 \times 10^{14}\ \mathrm{Hz} \end{aligned}$$

Calculate frequency corresponding to lower limit of wavelength range

Equation:

$$\begin{aligned} c &= f \times \lambda \\ f &= \frac{c}{\lambda} \\ &= \frac{3 \times 10^{8}\,\mathrm{m \cdot s^{-1}}}{500 \times 10^{-9}\,\mathrm{m}} \\ &= 6,00 \times 10^{14}\ \mathrm{Hz} \end{aligned}$$

Write final answer

The range of frequencies of green light is $5,31 \times 10^{14}\ \mathrm{Hz}$ to $6,00 \times 10^{14}\ \mathrm{Hz}$.

Calculating wavelengths and frequencies of light

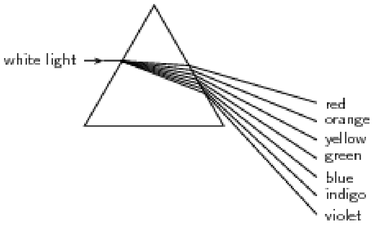
1. Calculate the frequency of light which has a wavelength of 400 nm. (Remember to use S.I. units)
2. Calculate the wavelength of light which has a frequency of 550×10^{12} Hz.
3. What colour is light which has a wavelength of 470×10^{-9} m and what is its frequency?
4. What is the wavelength of light with a frequency of 510×10^{12} Hz and what is its color?

Dispersion of white light

White light, like the light which comes from the sun, is made up of *all* the visible wavelengths of light. In other words, white light is a *combination* of all the colours of visible light.

You learnt that the speed of light is different in different substances. The speed of light in different substances depends on the frequency of the light. For example, when white light travels through glass, light of the different frequencies is slowed down by different amounts. The lower the frequency, the less the speed is reduced which means that red light (lowest frequency) is slowed down *less* than violet light (highest frequency). We can see this when white light is incident on a glass prism.

Have a look at the picture below. When the white light hits the edge of the prism, the light which travels through the glass is refracted as it moves from the less dense medium (air) to the more dense medium (glass).



- The **red** light which is slowed down the *least*, is refracted the *least*.
- The **violet** light which is slowed down the *most*, is refracted the *most*.

When the light hits the other side of the prism it is again refracted but the angle of the prism edge allows the light to remain separated into its different colours. White light is therefore separated into its different colours by the prism and we say that the white light has been **dispersed** by the prism.

The dispersion effect is also responsible for why we see rainbows. When sunlight hits drops of water in the atmosphere, the white light is dispersed into its different colours by the water.

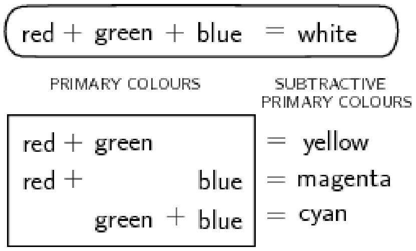
Addition and Subtraction of Light

Additive Primary Colours

The primary colours of light are **red**, **green** and **blue**. When all the primary colours are superposed (added together), white light is produced. Red, green and blue are therefore called the *additive primary colours*. All the other colours can be produced by different combinations of red, green and blue.

Subtractive Primary Colours

The subtractive primary colours are obtained by subtracting one of the three additive primary colours from white light. The subtractive primary colours are **yellow**, **magenta** and **cyan**. Magenta appears as a pinkish-purplish colour and cyan looks greenish-blue. You can see how the primary colours of light add up to the different subtractive colours in the illustration below.



Experiment : Colours of light

Aim:

To investigate the additive properties of colours and determine the complementary colours of light.

Apparatus:

You will need two battery operated torches with flat bulb fronts, a large piece of white paper, and some pieces of cellophane paper of the following colours: red, blue, green, yellow, cyan, magenta. (You should easily be able to get these from a newsagents.)

Make a table in your workbook like the one below:

Colour 1	**Colour 2**	**Final colour prediction**	**Final colour measured**
red	blue		
red	green		
green	blue		
magenta	green		
yellow	blue		
cyan	red		

Before you begin your experiment, use what you know about colours of light to write down in the third column "Final colour prediction", what you think the result of adding the two colours of light will be. You will then be able to test your predictions by making the following measurements:

Method:

Proceed according to the table above. Put the correct colour of cellophane paper over each torch bulb. e.g. the first test will be to put red cellophane on one torch and blue cellophane on the other. Switch on the torch with the red cellophane over it and shine it onto the piece of white paper.

What colour is the light?

Turn off that torch and turn on the one with blue cellophane and shine it onto the white paper.

What colour is the light?

Now shine both torches with their cellophane coverings onto the same spot on the white paper. What is the colour of the light produced? Write this down in the fourth column of your table.

Repeat the experiment for the other colours of cellophane so that you can complete your table.

Questions:

1. How did your predictions match up to your measurements?
2. Complementary colours of light are defined as the colours of light which, when added to one of the primary colours, produce white light. From your completed table, write down the complementary colours for red, blue and green.

Complementary Colours

Complementary colours are two colours of light which add together to give white.

Investigation : Complementary colours for red, green and blue

Complementary colours are two colours which add together to give white. Place a tick in the box where the colours in the first column added to the colours in the top row give white.

	magenta	**yellow**	**cyan**
	(=red+blue)	(=red+green)	(=blue+green)
red			
green			
blue			

You should have found that the complementary colours for red, green and blue are:

- Red and Cyan
- Green and Magenta
- Blue and Yellow

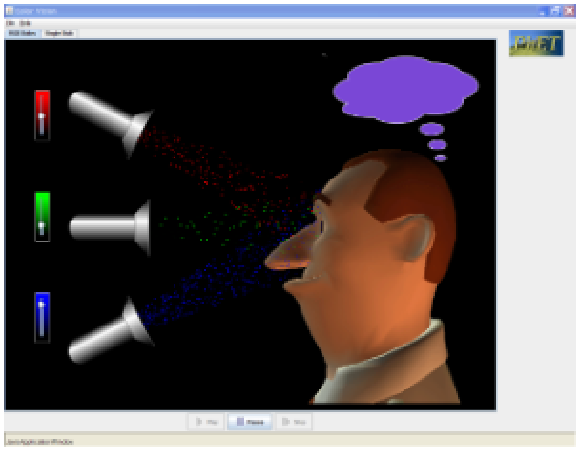
Perception of Colour

The light-sensitive lining on the back inside half of the human eye is called the retina. The retina contains two kinds of light sensitive cells or *photoreceptors*: the rod cells (sensitive to low light) and the cone cells (sensitive to normal daylight) which enable us to see. The rods are not sensitive to colour but work well in dimly lit conditions. This is why it is possible to see in a dark room, but it is hard to see any colours. Only your rods are sensitive to the low light levels and so you can only see in black, white and grey. The cones enable us to see colours. Normally, there are three kinds of cones, each containing a different pigment. The cones are activated when the pigments absorb light. The three types of cones are sensitive to (i.e. absorb) red, blue and green light respectively. Therefore we can perceive *all* the different colours in the visible spectrum when the different types of cones are stimulated by different amounts since they are just combinations of the three primary colours of light.

The rods and cones have different response times to light. The cones react quickly when bright light falls on them. The rods take a longer time to react. This is why it takes a while (about 10 minutes) for your eyes to adjust when you enter a dark room after being outside on a sunny day.

Note:
Interesting Fact
Color blindness in humans is the inability to perceive differences between some or all colors that other people can see. Most often it is a genetic problem, but may also occur because of eye, nerve, or brain damage, or due to exposure to certain chemicals. The most common forms of human color blindness result from problems with either the middle or long wavelength sensitive cone systems, and involve difficulties in discriminating reds, yellows,

and greens from one another. This is called "red-green color blindness". Other forms of color blindness are much rarer. They include problems in discriminating blues from yellows, and the rarest forms of all, complete color blindness or monochromasy, where one cannot distinguish any color from grey, as in a black-and-white movie or photograph.



run demo

Exercise:

Seeing Colours

Problem:

When blue and green light fall on an eye, is cyan light being created? Discuss.

Solution:

Think about what happens when light hits the eyes

Cyan light is not created when blue and green light fall on the eye. The blue and green receptors are stimulated to make the brain believe that cyan light is being created.

Colours on a Television Screen

If you look very closely at a colour cathode-ray television screen or cathode-ray computer screen, you will see that there are very many small red, green and blue dots called *phosphors* on it. These dots are caused to fluoresce (glow brightly) when a beam of electrons from the cathode-ray tube behind the screen hits them. Since different combinations of the three primary colours of light can produce any other colour, only red, green and blue dots are needed to make pictures containing all the colours of the visible spectrum.

Colours of light

1. List the three primary colours of light.
2. What is the term for the phenomenon whereby white light is split up into its different colours by a prism?
3. What is meant by the term “complementary colour” of light?
4. When white light strikes a prism which colour of light is refracted the most and which is refracted the least? Explain your answer in terms of the speed of light in a medium.

Pigments and Paints

We have learnt that white light is a combination of all the colours of the visible spectrum and that each colour of light is related to a different frequency. But what gives everyday objects around us their different colours?

Pigments are substances which give an object its colour by absorbing certain frequencies of light and reflecting other frequencies. For example, a red pigment absorbs all colours of light except red which it reflects. Paints and inks contain pigments which gives the paints and inks different colours.

Colour of opaque objects

Objects which you *cannot* see through (i.e. they are not transparent) are called **opaque**. Examples of some opaque objects are metals, wood and bricks. The colour of an opaque object is determined by the colours (therefore *frequencies*) of light which it *reflects*. For example, when white light strikes a blue opaque object such as a ruler, the ruler will absorb all frequencies of light *except* blue, which will be reflected. The reflected blue light is the light which makes it into our eyes and therefore the object will appear blue.

Opaque objects which appear white do not absorb any light. They reflect all the frequencies. Black opaque objects absorb all frequencies of light. They do not reflect at all and therefore appear to have no colour.

Exercise:
Colour of Opaque Objects

Problem:

If we shine white light on a sheet of paper that can only reflect green light, what is the colour of the paper?

Solution:

Think about what	Since the colour of an object is determined by that frequency of light that is *reflected*, the sheet of paper will appear green, as this is the only

determines colour	frequency that is reflected. All the other frequencies are absorbed by the paper.

Exercise:
Colour of an opaque object II

Problem:

The cover of a book appears to have a magenta colour. What colours of light does it reflect and what colours does it absorb?

Solution:

What colours make magenta?	We know that magenta is a combination of red and blue primary colours of light. Therefore the object must be reflecting blue and red light and absorb green.

Colour of transparent objects

If an object is **transparent** it means that you can see through it. For example, glass, clean water and some clear plastics are transparent. The colour of a transparent object is determined by the colours (frequencies) of light which it *transmits* (allows to pass through it). For example, a cup made of green glass will appear green because it absorbs all the other frequencies of light *except* green, which it transmits. This is the light which we receive in our eyes and the object appears green.

Exercise:
Colour of Transparent Objects

Problem:

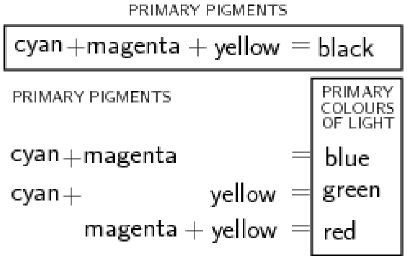
If white light is shone through a glass plate that absorbs light of all frequencies except red, what is the colour of the glass plate?

Solution:

Think about how light is transmitted	Since the colour of an object is determined by that frequency of light that is *transmitted*, the glass plate will appear red, as this is the only frequency that is not absorbed.

Pigment primary colours

The primary pigments and paints are **cyan**, **magenta** and **yellow**. When pigments or paints of these three colours are mixed together in equal amounts they produce **black**. Any other colour of paint can be made by mixing the primary pigments together in different quantities. The primary pigments are related to the primary colours of light in the following way:



PRIMARY PIGMENTS

cyan + magenta + yellow = black

PRIMARY PIGMENTS				PRIMARY COLOURS OF LIGHT
cyan +	magenta		=	blue
cyan +		yellow	=	green
	magenta +	yellow	=	red

Note:
Interesting Fact
Colour printers only use 4 colours of ink: cyan, magenta, yellow and black. All the other colours can be mixed from these!

Exercise:
Pigments

Problem: What colours of light are absorbed by a green pigment?

Solution:

Think about what colour is reflected

If the pigment is green, then green light must be *reflected*. Therefore, red and blue light are absorbed.

Exercise:
Primary pigments

Problem:

I have a ruler which reflects red light and absorbs all other colours of light. What colour does the ruler appear in white light? What primary pigments must have been mixed to make the pigment which gives the ruler its colour?

Solution:

What is being asked and what are we given?

We need to determine the colour of the ruler and the pigments which were mixed to make the colour.

An opaque object appears the colour of the light it reflects

The ruler reflects red light and absorbs all other colours. Therefore the ruler appears to be red.

What pigments need to be mixed to get red?

Red pigment is produced when magenta and yellow pigments are mixed. Therefore magenta and yellow pigments were mixed to make the red pigment which gives the ruler its colour.

Exercise:
Paint Colours

Problem:

If cyan light shines on a dress that contains a pigment that is capable of absorbing blue, what colour does the dress appear?

Solution:

Determine the component colours of cyan light

Cyan light is made up of blue and green light.

Determine solution

If the dress absorbs the blue light then the green light must be reflected, so the dress will appear green!

[missing_resource: http://static.slidesharecdn.com/swf/ssplayer2.swf?doc=colour-100511051146-phpapp01&stripped_title=colour-4047557&userName=kwarne]

End of Chapter Exercises

1. Calculate the wavelength of light which has a frequency of 570×10^{12} Hz.
2. Calculate the frequency of light which has a wavelength of 580 nm.
3. Complete the following sentence: When white light is dispersed by a prism, light of the colour ? is refracted the most and light of colour ? is refracted the least.
4. What are the two types of photoreceptor found in the retina of the human eye called and which type is sensitive to colours?
5. What color do the following shirts appear to the human eye when the lights in a room are turned off and the room is completely dark?
    1. red shirt
    2. blue shirt
    3. green shirt

6. Two light bulbs, each of a different colour, shine on a sheet of white paper. Each light bulb can be a primary colour of light - red, green, and blue. Depending on which primary colour of light is used, the paper will appear a different color. What colour will the paper appear if the lights are:
    1. red and blue?
    2. red and green?
    3. green and blue?
7. Match the primary colour of light on the left to its complementary colour on the right:

Column A	**Column B**
red	yellow
green	cyan
blue	magenta

8. Which combination of colours of light gives magenta?
    1. red and yellow
    2. green and red
    3. blue and cyan
    4. blue and red
9. Which combination of colours of light gives cyan?
    1. yellow and red
    2. green and blue
    3. blue and magenta
    4. blue and red
10. If yellow light falls on an object whose pigment absorbs green light, what colour will the object appear?
11. If yellow light falls on a blue pigment, what colour will it appear?

Electromagnetic Radiation

Introduction

This chapter will focus on the electromagnetic (EM) radiation. Electromagnetic radiation is a self-propagating wave in space with electric and magnetic components. These components oscillate at right angles to each other and to the direction of propagation, and are in phase with each other. Electromagnetic radiation is classified into types according to the frequency of the wave: these types include, in order of increasing frequency, radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays and gamma rays.

Particle/wave nature of electromagnetic radiation

If you watch a colony of ants walking up the wall, they look like a thin continuous black line. But as you look closer, you see that the line is made up of thousands of separated black ants.

Light and all other types of electromagnetic radiation seems like a continuous wave at first, but when one performs experiments with light, one can notice that light can have both wave and particle like properties. Just like the individual ants, the light can also be made up of individual bundles of energy, or quanta of light.

Light has both wave-like and particle-like properties (wave–particle duality), but only shows one or the other, depending on the kind of experiment we perform. A wave-type experiment shows the wave nature, and a particle-type experiment shows particle nature. One cannot test the wave and the particle nature at the same time. A particle of light is called a photon.

Photon

A photon is a quantum (energy packet) of light.

The particle nature of light can be demonstrated by the interaction of photons with matter. One way in which light interacts with matter is via the photoelectric effect, which will be studied in detail in [link].

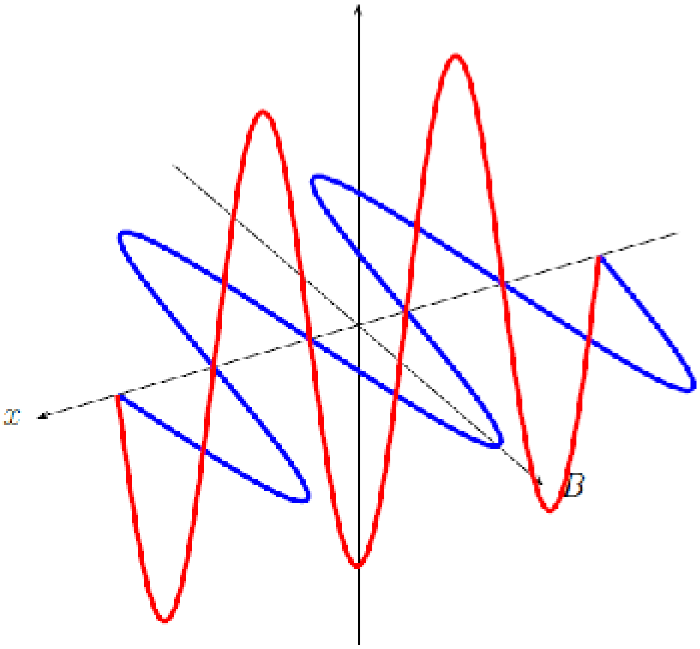
Particle/wave nature of electromagnetic radiation

1. Give examples of the behaviour of EM radiation which can best be explained using a wave model.
2. Give examples of the behaviour of EM radiation which can best be explained using a particle model.

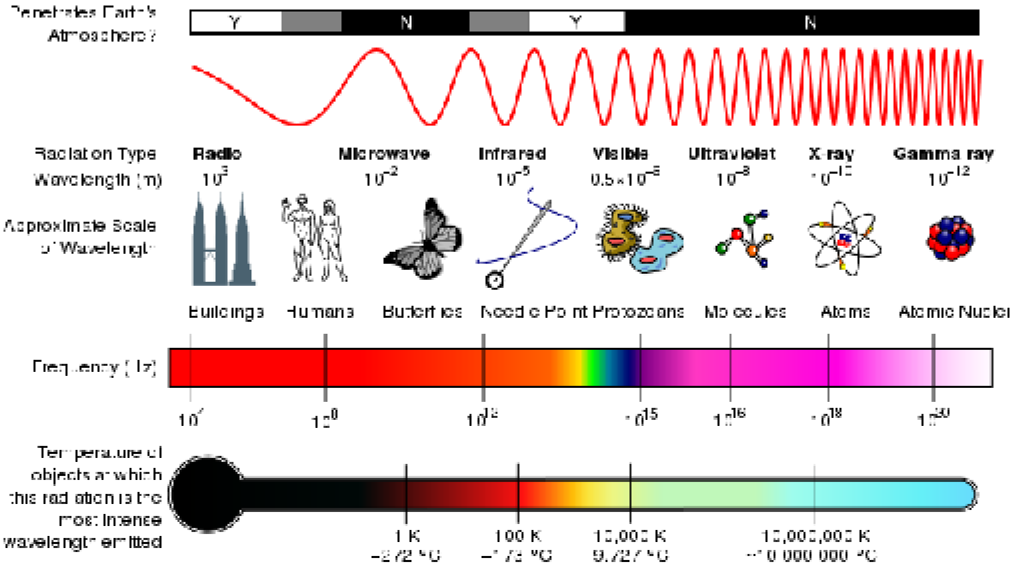
The wave nature of electromagnetic radiation

Accelerating charges emit electromagnetic waves. We have seen that a changing electric field generates a magnetic field and a changing magnetic field generates an electric field. This is the principle behind the propagation of electromagnetic waves, because electromagnetic waves, unlike sound waves, do not need a medium to travel through. EM waves propagate when an electric field oscillating in one plane produces a magnetic field oscillating in a plane at right angles to it, which produces an oscillating electric field, and so on. The propagation of electromagnetic waves can be described as *mutual induction*.

These mutually regenerating fields travel through empty space at a constant speed of $3 \times 10^{8}\,\text{m}\cdot\text{s}^{-1}$, represented by c.



Electromagnetic spectrum



The electromagnetic spectrum as a function of frequency. The different types according to wavelength are shown as well as everyday comparisons.

Observe the things around you, your friend sitting next to you, a large tree across the field. How is it that you are able to see these things? What is it that is leaving your friend's arm and entering your eye so that you can see his arm? It is light. The light originally comes from the sun, or possibly a light bulb or burning fire. In physics, light is given the more technical term electromagnetic radiation, which includes all forms of light, not just the form which you can see with your eyes.

Electromagnetic radiation allows us to observe the world around us. It is this radiation which reflects off of the objects around you and into your eye. The radiation your eye is sensitive to is only a small fraction of the total radiation emitted in the physical universe. All of the different fractions taped together make up the electromagnetic spectrum.

Dispersion

When white light is split into its component colours by a prism, you are looking at a portion of the electromagnetic spectrum.

The wavelength of a particular electromagnetic radiation will depend on how it was created.

Wave Nature of EM Radiation

1. List one source of electromagnetic waves. Hint: consider the spectrum diagram and look at the names we give to different wavelengths.
2. Explain how an EM wave propagates, with the aid of a diagram.
3. What is the speed of light? What symbol is used to refer to the speed of light? Does the speed of light change?
4. Do EM waves need a medium to travel through?

The radiation can take on any wavelength, which means that the spectrum is continuous. Physicists broke down this continuous band into sections. Each section is defined by how the radiation is created, not the wavelength of the radiation. But each category is continuous within the min and max wavelength of that category, meaning there are no wavelengths excluded within some range.

The spectrum is in order of wavelength, with the shortest wavelength at one end and the longest wavelength at the other. The spectrum is then broken down into categories as detailed in [link].

Category	**Range of Wavelengths (nm)**	**Range of Frequencies (Hz)**
gamma rays	<1	$> 3 \times 10^{19}$
X-rays	1-10	3×10^{17}-3×10^{19}
ultraviolet light	10-400	$7,5 \times 10^{14}$-3×10^{17}
visible light	400-700	$4,3 \times 10^{14}$-$7,5 \times 10^{14}$
infrared	700-10^5	3×10^{12}-$4,3 \times 10^{19}$
microwave	$10^5 - 10^8$	3×10^{9}-3×10^{12}
radio waves	$> 10^8$	$< 3 \times 10^{9}$

Electromagnetic spectrum

Since an electromagnetic wave is still a wave, the following equation that you learnt in Grade 10 still applies:

Equation:

$$c = f \cdot \lambda$$

Exercise:
EM spectrum I

Problem:

Calculate the frequency of red light with a wavelength of $4,2 \times 10^{-7}$ m

---

Solution:

Decide which formula to use and solve the problem

We use the formula: $c = f\lambda$ to calculate frequency. The speed of light is a constant 3×10^{8}m/s.

Equation:

$$\begin{aligned} c &= f\lambda \\ 3 \times 10^{8} &= f \times 4,2 \times 10^{-7} \\ f &= 7,14 \times 10^{14}\,\text{Hz} \end{aligned}$$

Exercise:
EM spectrum II

Problem:

Ultraviolet radiation has a wavelength of $200\ \text{nm}$. What is the frequency of the radiation?

Solution:

To calculate the frequency we need to identify the wavelength

Recall that all radiation travels at the speed of light (c) in vacuum. Since the question does not specify through what type of material the Ultraviolet radiation is traveling, one can assume that it is traveling through a vacuum. We can identify two

and the velocity of the radiation. properties of the radiation - $wavelength$ $(200\ \text{nm})$ and speed (c).

We can use the equation to find the frequency since the wavelength is given. **Equation:**

$$\begin{aligned} c &= f\lambda \\ 3\times 10^{8} &= f\times 200\times 10^{-9} \\ f &= 1.5\times 10^{15}\ \text{Hz} \end{aligned}$$

Examples of some uses of electromagnetic waves are shown in [link].

Category	**Uses**
gamma rays	used to kill the bacteria in marshmallows and to sterilise medical equipment
X-rays	used to image bone structures
ultraviolet light	bees can see into the ultraviolet because flowers stand out more clearly at this frequency
visible light	used by humans to observe the world
infrared	night vision, heat sensors, laser metal cutting
microwave	microwave ovens, radar
radio waves	radio, television broadcasts

Uses of EM waves

In theory the spectrum is infinite, although realistically we can only observe wavelengths from a few hundred kilometers to those of gamma rays due to experimental limitations.

Humans experience electromagnetic waves differently depending on their wavelength. Our eyes are sensitive to visible light while our skin is sensitive to infrared, and many wavelengths we do not detect at all.

EM Radiation

1. Arrange the following types of EM radiation in order of increasing frequency: infrared, X-rays, ultraviolet, visible, gamma.
2. Calculate the frequency of an EM wave with a wavelength of 400 nm.
3. Give an example of the use of each type of EM radiation, i.e. gamma rays, X-rays, ultraviolet light, visible light, infrared, microwave and radio and TV waves.

The particle nature of electromagnetic radiation

When we talk of electromagnetic radiation as a particle, we refer to photons, which are packets of energy. The energy of the photon is related to the wavelength of electromagnetic radiation according to:

Planck's constant

> Planck's constant is a physical constant named after Max Planck.
>
> $h = 6{,}626 \times 10^{-34}\ \text{J} \cdot \text{s}$

The energy of a photon can be calculated using the formula: $E = hf$ or $E = h\frac{c}{\lambda}$. Where E is the energy of the photon in joules (J), h is planck's constant, c is the speed of light, f is the frequency in hertz (Hz) and λ is the wavelength in metres (m).

Exercise:
Calculating the energy of a photon I

Problem:

Calculate the energy of a photon with a frequency of 3×10^{18} Hz

---

Solution:

We use the formula:

Equation:

$$\begin{aligned} E &= hf \\ &= 6,6 \times 10^{-34} \times 3 \times 10^{18} \\ &= 2 \times 10^{-15}\,\mathrm{J} \end{aligned}$$

Exercise:
Calculating the energy of a photon II

Problem:

What is the energy of an ultraviolet photon with a wavelength of 200 nm?

Solution:

Determine what is required and how to approach the problem.

We are required to calculate the energy associated with a photon of ultraviolet light with a wavelength of 200 nm.

Equation:

We can use:

$$E = h\frac{c}{\lambda}$$

Solve the problem

Equation:

$$\begin{aligned} E &= h\frac{c}{\lambda} \\ &= \left(6,626 \times 10^{-34}\right)\frac{3 \times 10^{8}}{200 \times 10^{-9}} \\ &= 9,939 \times 10^{-10}\,\text{J} \end{aligned}$$

Exercise - particle nature of EM waves

1. How is the energy of a photon related to its frequency and wavelength?
2. Calculate the energy of a photon of EM radiation with a frequency of 10^{12} Hz.
3. Determine the energy of a photon of EM radiation with a wavelength of 600 nm.

Penetrating ability of electromagnetic radiation

Different kinds of electromagnetic radiation have different penetrabilities. For example, if we take the human body as the object. Infrared light is emitted by the human body. Visible light is reflected off the surface of the human body, ultra-violet light (from sunlight) damages the skin, but X-rays are able to penetrate the skin and bone and allow for pictures of the inside of the human body to be taken.

If we compare the energy of visible light to the energy of X-rays, we find that X-rays have a much higher energy. Usually, kinds of electromagnetic radiation with higher energy have higher penetrabilities than those with low energies.

Certain kinds of electromagnetic radiation such as ultra-violet radiation, X-rays and gamma rays are very dangerous. Radiation such as these are called ionising radiation. Ionising radiation transfers energy as it passes through matter, breaking molecular bonds and creating ions.

Excessive exposure to radiation, including sunlight, X-rays and all nuclear radiation, can cause destruction of biological tissue.

Ultraviolet(UV) radiation and the skin

UVA and UVB are different ranges of frequencies for ultraviolet (UV) light. UVA and UVB can damage collagen fibres which results in the speeding up skin aging. In general, UVA is the least harmful, but it can contribute to the aging of skin, DNA damage and possibly skin cancer. It penetrates deeply and does not cause sunburn. Because it does not cause reddening of the skin (erythema) it cannot be measured in the SPF testing. There is no good clinical measurement of the blocking of UVA radiation, but it is important that sunscreen block both UVA and UVB.

UVB light can cause skin cancer. The radiation excites DNA molecules in skin cells, resulting in possible mutations, which can cause cancer. This cancer connection is one reason for concern about ozone depletion and the ozone hole.

As a defense against UV radiation, the body tans when exposed to moderate (depending on skin type) levels of radiation by releasing the brown pigment melanin. This helps to block UV penetration and prevent damage to the vulnerable skin tissues deeper down. Suntan lotion, often referred to as sunblock or sunscreen, partly blocks UV and is widely available. Most of these products contain an SPF rating that describes the amount of protection given. This protection, however, applies only to UVB rays responsible for sunburn and not to UVA rays that penetrate more deeply into the skin and may also be responsible for causing cancer and wrinkles. Some sunscreen lotion now includes compounds such as titanium dioxide which helps protect against UVA rays. Other UVA blocking compounds found in sunscreen include zinc oxide and avobenzone.

What makes a good sunscreen?

- UVB protection: Padimate O, Homosalate, Octisalate (octyl salicylate), Octinoxate (octyl methoxycinnamate)
- UVA protection: Avobenzone
- UVA/UVB protection: Octocrylene, titanium dioxide, zinc oxide, Mexoryl (ecamsule)

Another means to block UV is by wearing sun protective clothing. This is clothing that has a UPF rating that describes the protection given against both UVA and UVB.

Ultraviolet radiation and the eyes

High intensities of UVB light are hazardous to the eyes, and exposure can cause welder's flash (photo keratitis or arc eye) and may lead to cataracts, pterygium and pinguecula formation.

Protective eyewear is beneficial to those who are working with or those who might be exposed to ultraviolet radiation, particularly short wave UV. Given that light may reach the eye from the sides, full coverage eye protection is usually warranted if there is an increased risk of exposure, as in high altitude mountaineering. Mountaineers are exposed to higher than ordinary levels of UV radiation, both because there is less atmospheric filtering and because of reflection from snow and ice.

Ordinary, untreated eyeglasses give some protection. Most plastic lenses give more protection than glass lenses. Some plastic lens materials, such as polycarbonate, block most UV. There are protective treatments available for eyeglass lenses that need it which will give better protection. But even a treatment that completely blocks UV will not protect the eye from light that arrives around the lens. To convince yourself of the potential dangers of stray UV light, cover your lenses with something opaque, like aluminum foil, stand next to a bright light, and consider how much light you see, despite the complete blockage of the lenses. Most contact lenses help to protect the retina by absorbing UV radiation.

X-rays

While x-rays are used significantly in medicine, prolonged exposure to X-rays can lead to cell damage and cancer.

For example, a mammogram is an x-ray of the human breast to detect breast cancer, but if a woman starts having regular mammograms when she is too young, her chances of getting breast cancer increases.

Gamma-rays

Due to the high energy of gamma-rays, they are able to cause serious damage when absorbed by living cells.

Gamma-rays are not stopped by the skin and can induce DNA alteration by interfering with the genetic material of the cell. DNA double-strand breaks are generally accepted to be the most biologically significant lesion by which ionising radiation causes cancer and hereditary disease.

A study done on Russian nuclear workers exposed to external whole-body gamma-radiation at high cumulative doses shows a link between radiation exposure and death from leukaemia, lung, liver, skeletal and other solid cancers.

Cellphones and electromagnetic radiation

Cellphone radiation and health concerns have been raised, especially following the enormous increase in the use of wireless mobile telephony throughout the world. This is because mobile phones use electromagnetic waves in the microwave range. These concerns have induced a large body of research. Concerns about effects on health have also been raised regarding other digital wireless systems, such as data communication networks. In 2009 the World Health Organisation announced that they have found a link between brain cancer and cellphones.

Cellphone users are recommended to minimise radiation, by for example:

1. Use hands-free to decrease the radiation to the head.
2. Keep the mobile phone away from the body.
3. Do not telephone in a car without an external antenna.

Exercise - Penetrating ability of EM radiation

1. Indicate the penetrating ability of the different kinds of EM radiation and relate it to energy of the radiation.
2. Describe the dangers of gamma rays, X-rays and the damaging effect of ultra-violet radiation on skin

[missing_resource: http://static.slidesharecdn.com/swf/ssplayer2.swf?doc=electromagneticspectrum-100512094547-phpapp02&stripped_title=electromagnetic-spectrum-4069478&userName=kwarne]

Summary

1. Electromagnetic radiation has both a wave and a particle nature.
2. Electromagnetic waves travel at a speed of $3 \times 10^8\ m \cdot s^{-1}$ in a vaccum.
3. The Electromagnetic spectrum consists of the follwing types of radiation: radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma-rays.
4. Gamma-rays have the most energy and are the most penetrating, while radio waves have the lowest energy and are the least penetrating.

End of chapter exercise

1. What is the energy of a photon of EM radiation with a frequency of 3×10^8 Hz?
2. What is the energy of a photon of light with a wavelength of 660 nm?

3. List the main types of electromagnetic radiation in order of increasing wavelength.
4. List the main uses of:

    1. radio waves
    2. infrared
    3. gamma rays
    4. X-rays

5. Explain why we need to protect ourselves from ultraviolet radiation from the Sun.
6. List some advantages and disadvantages of using X-rays.
7. What precautions should we take when using cell phones?
8. Write a short essay on a type of electromagnetic waves. You should look at uses, advantages and disadvantages of your chosen radiation.
9. Explain why some types of electromagnetic radiation are more penetrating than others.